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ORIGINAL ARTICLE

Low numeracy and dyscalculia: identification and intervention

Brian Butterworth · Diana Laurillard

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Abstract One important factor in the failure to learn arithmetic in the normal way is an endogenous core deficit in the sense of number. This has been associated with low numeracy in general (e.g. Halberda et al. in Nature 455:665–668, 2008) and with dyscalculia more specifically (e.g. Landerl et al. in Cognition 93:99–125, 2004). Here, we describe straightforward ways of identifying this deficit, and offer some new ways of strengthening the sense of number using learning technologies.

1 Introduction

One of the central problems in educational neuroscience is how to coordinate the disciplines of education and neuroscience. As (Varma, McCandliss, and Schwartz, 2008) note, this could be a long job. At the new Centre for Educational Neuroscience in London, an inter-institutional development involving UCL (University College London), the Institute of Education and Birkbeck College, both of the University of London, we see the way forward consisting in a continuing conversation among neuroscientists, psychologists and educators, seeking issues of common concern, agreed methods and useful applications. In this paper, we report on a domain-specific project to investigate the neuroscience of number and how to relate these findings to their implications for mathematical education.

Perhaps the most important difficulty we face is how to evaluate the results of our research. In neuroscience and psychology, we can design an experiment, run it and get a result, which is methodologically straightforward. However, education is far more complex, and getting a "result" is not straightforward, due to the interference of the many variables in the learning environment.

In this study, instead of attempting a large-scale summative evaluation of our interventions for dyscalculia, we are at this stage conducting an intensive *formative evaluation* of our methods, to look at the detailed effects on the behaviour of selected individuals, in the context of specific task activities and number concepts. The aim is to provide data on each learner's difficulties, their behaviour on the learning activities, and the changes in their performance within the task, and thereby develop an account of the role the intervention plays, if any, for each learner's conceptual development.

The other important issue in educational neuroscience is to close the circle from neuroscience to education and back again to the neuroscience. We have just begun this process and can only indicate how this might continue in the future.

In this paper, our first aim is to show how we get from neuroscience to an analysis of the core cognitive problem in dyscalculia. However, even if one can reliably identify the core problem—behaviourally, neurally, or genetically, and we have indicators for all of these—this does not uniquely determine the form the pedagogic intervention should take. This is a separate process. The science *informs* the design of the intervention, as the science of materials informs, but does not determine the design of a bridge. And as in engineering, where theory is silent, the design can

B. Butterworth (✉)
Institute for Cognitive Neuroscience, University College London, 17 Queen Square, London WC1N 3AR, UK
e-mail: b.butterworth@ucl.ac.uk

B. Butterworth · D. Laurillard
Centre for Educational Neuroscience, London, UK

D. Laurillard
London Knowledge Lab, Institute of Education University of London, London, UK

also build on current best practice, in this case from pedagogic theory, Special Educational Needs (SEN) teachers, and research in mathematical education.

Our broad interdisciplinary strategy, as applied to dyscalculia, is captured in Fig. 1. The process of developing an intervention involves continuously iterated interactions within and among the disciplines with the aim of modifying the learner's behaviour and cognition. We have just begun to use the results from neuroscience to inform pedagogic design, so this is represented by a strong dotted line. The effects of interventions on neural structure and activity are currently unclear, hence the weak dotted line, but it is part of our research agenda to measure the neural effects of intervention.

1.1 The biological basis of numeracy and dyscalculia

It is well-established from neuroimaging studies that the critical area for processing numbers in the human lies in the intraparietal sulcus (IPS), with an impressive body of work showing that this area of the brain deals with the comparison of digits and of dot arrays (Pinel, Dehaene, Rivière, and Le Bihan, 2001; Tang, Critchley, Glaser, Dolan, and Butterworth, 2006) and with enumeration of objects (Castelli, Glaser, and Butterworth, 2006; Piazza, Mechelli, Price, and Butterworth, 2006). These fundamental functions underpin the normal development of arithmetical skills (Butterworth, 2005b), and the parietal lobes are deeply implicated in the neural network for more complex calculations (e.g. Dehaene, Spelke, Pinel, Stanescu, and Tsivkin, 1999; Zago et al., 2001), and even in an extended network that sustains the calculations of prodigies (Butterworth, 2001; Pesenti et al., 2001).

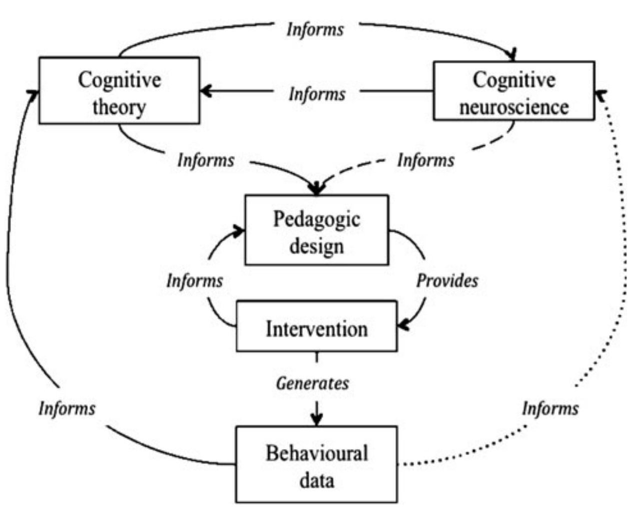

Fig. 1 Interdisciplinary strategy in the Centre for Educational Neuroscience. *Solid lines* indicate established interdisciplinary connections. *Dashed line* represents the contribution cognitive neuroscience makes in this study on pedagogic design. The *dotted line* indicates the way in which the behavioural data, in the form of neural changes resulting from intervention, can now be used to inform the neuroscience

The essential role of the parietal areas in numerical processing and calculation has been demonstrated by observations of neurological patients in whom these areas have been damaged. A case series study by Henschen (1920) showed that left parietal damage led to problems with numbers, often with no other cognitive symptoms. More recently, detailed case studies have shown that damage to the left parietal lobe can lead to severe acalculia with other cognitive functions intact (Cipolotti, Butterworth, & Denes, 1991), while the contrasting pattern has been reported in a patient with relatively intact parietal lobes but with progressive atrophy in the temporal lobes. This patient had very severe deficits of language and semantic memory, although their calculation was intact, even for long divisions and multiplications, when presented in written form (Cappelletti, Butterworth, & Kopelman, 2001; Cappelletti, Kopelman, & Butterworth, 2002). Converging evidence of the critical role of the parietal lobes has been demonstrated using transcranial magnetic stimulation, which has been found to disrupt basic numerical processing, such as number comparison (Cappelletti, Barth, Fregni, Spelke, & Pascuale-Leone, 2007; Cohen Kadosh et al., 2007), and also arithmetical fact retrieval (Rusconi, Walsh, and Butterworth, 2005).

The differential contributions of right and left parietal lobes have not yet been convincingly demonstrated. In a case series study, Warrington and James (1967) found that patients with right parietal damage were impaired on estimating the number of dots in a visual array. This function of the right parietal has been supported by several recent neuroimaging studies, including one showing selective activation in a sequential estimation task (Piazza et al., 2006). Evidence for this has been provided by a comparative study of neural activations in children and adults, showing much more left lateralization in adults (Cantlon, Brannon, Carter, and Pelphrey, 2006).

It has been found that individuals with developmental disorders of low numeracy or dyscalculia have abnormalities in the parietal lobe. Isaacs, Edmonds, Lucas, & Gadian (2001) found a reduced grey matter in the left IPS as compared with controls matched by age and birth history. Even though these individuals were cognitively normal, this was an untypical sample, consisting of adolescents with very low birth weight. Evidence for left parietal abnormality in association with poor numerical abilities comes from studies of females with Turner's syndrome (Molko et al., 2003). Again the findings may not be representative of the neural basis of learners with low numeracy who have not been clinically referred.

More relevant are differences in neural activations between dyscalculics and controls on a very simple number comparison task. Normally, activation in the parietal lobe, especially in the IPS, is modulated by the difference between the numbers to be compared. It has long been known that reaction time and errors, even for single-digit comparison, depend on the difference, usually termed the "distance", between the two numbers, such that the larger the distance, the easier the judgment (Moyer & Landauer, 1967). In number comparison tasks, the smaller the distance the greater the activation, whether stimuli are digits or dots (Pinel et al., 2001; Tang et al., 2006). However, this neural activation distance effect is not present in the brains of 11-year-old dyscalculic children, suggesting abnormal mental processing of numerical magnitude (Price, Holloway, Räsänen, Vesterinen, & Ansari, 2007).

An important aspect of the study by Price et al. (2007) is that the abnormal activation pattern was found in the right IPS not in the left. Right IPS structural differences in 9-year-old children have also been observed (Rotzer et al., 2008). On the other hand, *bilateral* IPS structural abnormalities have also been reported in 7–9-year-old children (Rykhlevskaia, Uddin, Kondos, & Menon, 2009), and also activation differences in 9–11-year-olds (Mussolin et al., 2009). It seems likely at this time that the adult pattern of a bilateral network may emerge over the course of development (Ansari & Karmiloff-Smith, 2002); indeed, it has been suggested by Piazza et al. (2006) that basic numerosity processing begins in the right IPS, and then is copied into the left IPS to link up with language processing, perhaps due to the functionally critical role of verbal counting in the development of arithmetic (Butterworth, 2005b). Thus, the critical role of left IPS and the left angular gyrus in adult *calculation* could reflect this developmental trajectory.

More generally, it has been argued that the *core deficit* in dyscalculia, but not necessarily in other forms of low numeracy, is an inability to process numerosities, which are properties of sets. As in Fig. 2, the numerosity processing brain areas are part of the calculation network. Counting and manipulating sets are the way most of us learn arithmetic (Butterworth, 2005a). Dyscalculics are poor in performance, either in accuracy or in time, on very simple numerosity tasks, such as number comparison or the enumeration of small arrays of objects (Butterworth & Reigosa Crespo, 2007; Landerl, Bevan, & Butterworth, 2004), see Fig. 3.

In one study, we compared four groups of 9-year-olds, matched by age and IQ, on tests of dot enumeration (up to 9 dots) and number comparison (select the larger number). The first group, the dyscalculics, were at least 3SDs worse than controls on timed single-digit addition, and were also identified by their teachers as having particular problems learning arithmetic. The second group tested were dyslexics, to determine whether dyslexia, or its underlying

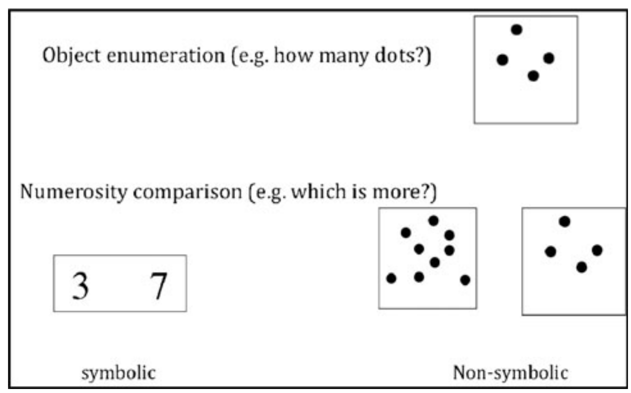

Fig. 3 Numerosity tasks on the enumeration of dot arrays and the comparison of digits and of dot arrays

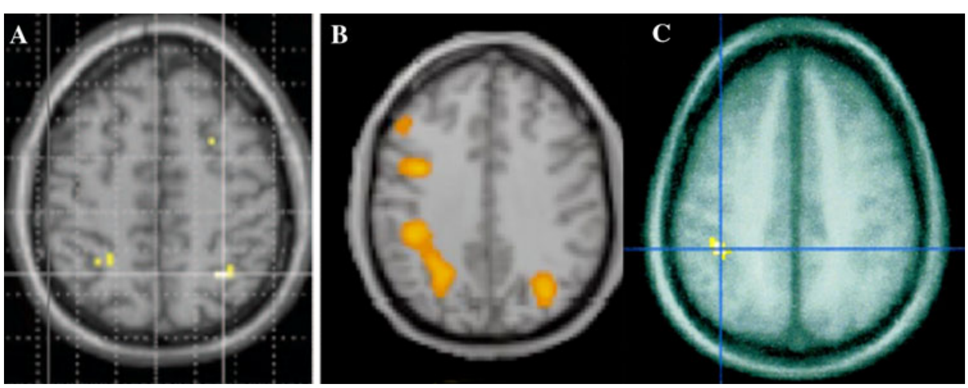

Fig. 2 Normal and atypical adult brain areas for bilateral number processing in the intraparietal sulcus. **a** The *highlighted parts* show the areas that are normally activated in numerosity comparison tasks (Castelli et al., 2006). **b** The *highlighted parts* show the networks normally activated for arithmetical calculation, which include the numerosity processing areas (Zago et al., 2001). **c** The *highlight* indicates the part that is found to be structurally abnormal in adolescent dyscalculics (Isaacs et al., 2001)

phonological deficit, in itself had a detrimental effect on numerosity abilities. The third group included both dyscalculic and dyslexic learners on these criteria to see whether dyslexia contributed to a more adverse outcome than dyscalculia alone. Finally, there was a group of matched controls from the same classes as the experimental groups. Briefly, we found that dyscalculics were significantly worse on the two tests of simple numerosity processing: dot enumeration and number comparison. The dyslexics performed at the same level as controls and the double-deficit group performed at the same level as the dyscalculics (Landerl et al., 2004). This suggests that the ability to deal with numerosity is defective in dyscalculics. Further research by other teams also supports the importance of numerosity abilities in acquiring arithmetical skills (Halberda, Mazzocco, & Feigenson, 2008; Iuculano, Tang, Hall, & Butterworth, 2008; Koontz & Berch, 1996).

We have produced a computerized test based on this approach, standardized against UK norms for assessing learners from 6 to 14 years. This test, the *Dyscalculia Screener*, runs on a PC, and uses keyboard responses. It comprises four subtests:

1. dot enumeration;
2. number comparison;
3. single-digit arithmetic (addition only for younger children, plus multiplication for older children);
4. simple reaction time.

The two basic capacity tests, and the attainment test for arithmetic are all item-timed, so that both accuracy and speed can be assessed. In fact, the software computes an efficiency measure that combines the two. However, since each test is timed, it is important to distinguish general slowness or quickness in making decisions and responses, from those specific to numerical tasks. For this reason, the simple reaction time test is also included. The screener can be used to identify dyscalculics as those learners who perform below the 7th percentile on the capacity tests. It is quite widely used in UK schools.

There is evidence that the numerosity processing is an inherited capacity, one that has deep evolutionary roots (Agrillo, Dadda, Serena, & Bisazza, 2009; Ansari, 2008; Butterworth, 2000; Dehaene, Molko, & Cohen, 2004; Feigenson, Dehaene, & Spelke, 2004). Therefore, a deficit in this capacity may also be heritable. There is evidence from twin studies that this is indeed the case: identical twins being more similar in their numerical abilities than non-identical twins (Alarcon, Defries, Gillis Light, & Pennington, 1997). Siblings of dyscalculics are more likely to be dyscalculic than siblings of non-dyscalculics, indeed about 15 times more likely (Shalev et al., 2001). Certain genetic abnormalities have detrimental effects on numerosity processing. For example, it has been found that females with Turner's syndrome perform poorly on number comparison and enumeration (Bruandet, Molko, Cohen, & Dehaene, 2004; Butterworth et al., 1999).

This does not mean, of course, that all dyscalculics have inherited the condition, and important sources of variation are both common and unique environmental influences (Kovas, Haworth, Dale, and Plomin, 2007), and it is known that pre-term birth and early nutrition can have long-term cognitive effects (Isaacs et al., 2008).

It is also clear that general cognitive factors can affect learning arithmetic (Ansari and Karmiloff-Smith, 2002). Individual differences in working memory (WM) have been related to individual differences in arithmetical attainment in school (Gathercole & Pickering, 2000). Children with specific difficulties in arithmetic as compared with both age-matched and ability-matched controls do not differ on tasks that tap the phonological loop (PL) component, such as immediate serial recall (e.g. digit span) (e.g. McLean & Hitch, 1999); indeed, learners with normal span can be dyscalculic (Landerl et al., 2004). However, tasks tapping the central executive (CE) component are sometimes associated with poor arithmetic (e.g. McLean & Hitch, 1999). Indeed, both components of the system could play a role in the typical or atypical development of numerical cognition, but at distinct stages of development. The PL could contribute to the development of the number vocabulary and the counting strings, while the CE could be responsible of keeping track of the mental operations (Noël, Seron, & Trovarelli, 2004). However, Geary et al. (2009), in an extensive longitudinal study, did not find that CE was a significant latent factor, but did find that tests of basic numerical capacity, such as number line and set tasks, were the most important predictors of arithmetical development.

2 Bridging from neuroscience to education

Neuroscience research tells us, therefore, that for some learners there is a core deficit of numerosity processing, indexed by dot enumeration or by magnitude comparison. However, these findings do not in themselves determine the pedagogical intervention needed to assist such learners. They establish that processes such as dot enumeration are fundamental to an understanding of number, and therefore of arithmetic, and suggest that pedagogical interventions should aim to ensure that all learners develop the ability to enumerate dot patterns by some means, if they are to progress to understanding basic arithmetic.

To take these outcomes further into making an impact in the classroom, we need a principled way of developing the optimal pedagogy, informed by the neuroscience, which can lead to improved performance by the children.

Evaluation is not straightforward in the normal classroom context. Summative evaluation is complex because there are many sources of variation in learner performance, so that either a massive effect size or a very large sample is needed to demonstrate a result. Developing an abstract concept, such as the complementarity of addition and subtraction ($a + b = c \rightarrow c - a = b$), is a process that necessarily takes place over an elapsed time that goes well beyond the controlled experimental session. Even with frequent testing, the process is affected by many extraneous and uncontrollable variables external to the learning environment, such as family environment, health status, nutrition, and the immediate physical and social aspects of the school environment itself.

In a study such as this, where we are attempting to discover the pedagogy that works for learners with the most difficulties, there is also the further complication that we need *formative evaluation of the teaching method*, an iterative design process by which the method is improved on the basis of success evaluations. This formative evaluation must take into account the progress of the learners using the method, and also the informed judgment of the teachers.

If, eventually, a successful pedagogy achieves a demonstrable benefit for the learners, in terms of improved performance over a shorter time than in the normal classroom, then there will be an opportunity for comparative neural imaging data, before and after the pedagogical intervention that could show differences in a learner's neural processing for specific tasks and specific initial deficits. This form of summative evaluation, of individual learner cases showing small effects over long time periods, would be more appropriate for this kind of research than large sample studies, where the effects would be lost among the many other context effects on learning. However, at this stage in the research, the task is still to discover the successful pedagogies.

3 Digital intervention

There is an extensive literature on more traditional ways to help low-attaining learners of arithmetic, and we do not have the space to discuss it in detail here. A useful review for the UK government of intervention schemes actually used in UK schools is Dowker (2009). "If there is no assessment, or no adequate guidance as to carrying out assessments, then it is much more difficult to carry out targeted interventions", she writes (page 17). She suggests that interventions have to be personalized for the learner's particular pattern of cognitive strengths and weaknesses. See also articles in (Dowker, 2008). Theoretically motivated interventions have been proposed and tested in carefully controlled but typically smaller scale studies (e.g. Griffin, Case, & Siegler, 1994).

The particular form of pedagogical intervention chosen for this study relies on digital technologies rather than teacher–student small group instruction for learners with special needs, which is already in use in schools. There were several reasons for this. Digital intervention programs specially designed for learners with dyscalculia and low numeracy offer distinctive benefits for learners and teachers, and for the research.

3.1 Benefits for learners

- *Practice-oriented*: Digital programs provide the opportunity for unsupervised repeated practice, as they are designed to be easy for learners to use alone, once they have been introduced. They can be downloaded to the family's home computer, and could potentially be put onto new generation mobile phones for learners who have their own. A simple interface design makes this feasible.
- *Age-independent*: Simple interface designs also mean that programs can be age-independent, and can be used as well for adult learners with numeracy problems. Objects used in the programs can be stored as interchangeable picture files, so that programs, unlike print materials, can be customized to the age group.
- *Needs-oriented*: Manipulation of digital objects, using a mouse or touch screen, is easier than physical objects, such as handfuls of cards, for learners who may often have slight dyspraxia as well as dyscalculia. Simple audio instructions can replace the text of paper-based exercises, for learners who may also have reading difficulties
- *Meaningful*: Virtual environments can link the physical to the abstract in ways that are not possible in the physical world, e.g. the learner can 'zoom into' a 1–10 number line to discover decimal numbers.
- *Private*: Digital programs are endlessly patient, offer no threat, and are private encounters, providing feedback to the learner without the involvement of anyone else; these properties are extremely valuable for learners who struggle to keep up, and repeatedly suffer defeats in normal classroom contexts.

3.2 Benefits for teachers

- *Customizable*: The pedagogy embedded in digital programs takes the form of specific parameterized rules for generating the next task on the basis of current learner performance. The parameters used in these rules (such as how quickly to introduce new dot patterns) can be exposed for the teacher to control, if they wish. Similarly, they can select from a range of dot pattern pictures to match those they are using in the classroom (there are several versions in use).

- *Shareable*: A digital program captures the pedagogy of SEN teaching in a way that can be shared and trialled by SEN teachers in the context of an *online community of practice*, exposing the current best practice to their collective critique. Many SEN teachers find they are relatively isolated in their school and welcome contact with others facing, and sometimes solving the same teaching challenges.
- *Personalized*: One-to-one tuition is essential for SEN learners. A well-designed program can provide supervised individual learning for some learners, enabling the teacher to give their undivided attention to another learner.
- *Motivating*: Homework is difficult to manage for learners who are struggling, and the home environment may not be conducive to doing the much-needed repeated practice—an interactive and adaptive program, if accessible, is much more likely to achieve this than paper-based tasks.

3.3 Benefits for research

- *Consistency*: The pedagogy design is captured digitally, and described as a particular set of tasks, sequenced according to explicit rules, so that the form of the intervention is known exactly and can be reliably shared with others, unlike human interventions, which can vary considerably from one teacher to another.
- *Automated*: The programs collect and format learner data automatically.
- *Progress tracking*: As the learner is moved from one task to another, the digital programs provide the opportunity to examine the transfer of learning at the level of accuracy and RTs on carefully sequenced learning tasks.

Explicit description of the pedagogy is important for this kind of research, although it is not always a focus of evaluations. Studies of the effectiveness of digital technologies for learning often show only small positive effects, leading reviewers to generalizations such as "the results of these six studies suggest that computer simulations can sometimes be used to improve the effectiveness of science teaching" (Kulik, 2003, p. 59), or "CAI [Computer-Assisted Instruction] is effective in math" (Slavin & Lake, 2008, p. 481). However, here, there is no recognition of the interaction between the pedagogy and the medium. From a set of controlled studies of particular instantiations of a teaching medium, it is not possible to draw conclusions about the medium in general: we would not draw such sweeping conclusions about lectures or books from six controlled studies and their particular instantiations. The nature of the pedagogic design is the most critical factor in determining effectiveness, but these studies do not describe the detail of the design, only that it is delivered in a particular medium. Of course, it is *possible* for CAI, or simulations, or other digital formats, to be effective, but we need better accounts of the nature of the pedagogy that works, if we are to build on these studies.

Reducing sources of variance is aided by the use of a standardized program, in several ways:

- *Aligning the teacher's intention with the learner's*: SEN teachers frequently use games with their groups, because these learners will often focus their attention on pleasing the teacher, rather than the task at hand. A game creates a shared goal, improving the likelihood that the learner is carrying out the intended task.
- *Quality of teaching*: for good or ill, the digital program works the same way for all learners being studied.
- *Learner distractibility*: the digital program is not immune from this in the class environment, but does tend to engage the learner's attention for a longer period because it is always waiting for an input, and always responds immediately to what the learner does.
- *Human relationships*: teacher–learner and learner–learner relationships introduce many sources of variance in the learning process, whereas a digital program is unvarying in what it does, making the learner the primary focus of the variation generated.

Digital interventions have the drawback that they are difficult and expensive to develop, but once a successful format is developed it is widely replicable in a way that human interventions cannot be.

4 Developing a pedagogy informed by cognitive neuroscience

To develop an effective pedagogy, we use a three-pronged approach: (a) using the neuroscience to determine the types of tasks and performance that provide evidence of normal numerosity processing; (b) using expert SEN teachers' experience and practice to inform the presentation and sequencing of these tasks; and (iii) using theoretical pedagogical principles to generate the way the learner interacts with the task goals, actions and feedback. The digital interventions are designed on this basis:

- *Neuroscience findings*: Use tasks that exercise dot enumeration and that generate behavioural data comparable with the neuroscience tasks

- *SEN teaching approaches*: Use existing tried and tested tasks, published in the SEN literature, which emphasize a focus on numerosity, number comparisons, number bonds, and embedding the mathematical tasks within game-like tasks and situations.
- *Pedagogical principles*: Use tasks based on a 'constructionist' approach (Healy & Kynigos, 2010; Noss & Hoyles, 1996), which situates the learner within an environment that affords learning in the sense that it: makes the task goal a meaningful goal for the learner; provides the means for them to act to achieve that goal; feeds back the result of their action in relation to the goal; and motivates revision of the action to improve the fit to the goal.

A further design principle is adaptivity, i.e. to emulate the good teacher who adapts the task to the learner's needs. Our aim is to provide software that adapts to current learner understanding and maintains the learner in the zone of proximal development (Vygotsky, 1978), as a teacher does, to consolidate their learning but also to keep the tasks challenging and progressively more difficult, to build the concept (Mariotti, 2009). To do this, the programs embody the following adaptivity rules:

Time-based rule (to check whether performance is fluent):

- If the target reaction time has been reached for correctly selecting number N for this level, then reduce the probability that N will be selected as a trial.
- If the reaction time for N is less than the current required reaction time for N, then reduce this time for number N by 1 s.
- Otherwise, if the reaction time for N is greater than the current required reaction time, then increase this time for number N by 1 s.
- Continue trials until current required reaction time is the same as the target for that number at that level.

Construction rule (to support development of performance):

- If the input for number N is incorrect then show the result, and enable construction of correct result.

Iteration rule (to support extension of performance):

- If the input for number N is correct then show the result, and increase the score for N, and if the score for $N =$ the target score for N for this level, then reduce the probability that N will be selected for a trial, and add a new number item to the selection bank.

The time-based rule adapts to the learner's pace, but adjusts the difficulty of the task as they improve. The construction rule provides goal-oriented action with feedback that is meaningful in the sense that the learner is able to adjust their action to achieve the goal (see Sect. 5). The iteration rule adapts to the learner's increasing knowledge by rehearsing known concepts less while introducing new ones. The default parameters for these rules, such as the target score and the initial target reaction time, are displayed in the 'Teacher Preferences' section of the program, with simple ways for the teacher to change them. Task designs are based on the learning activities used in class work by experienced special needs teachers.

The environment is, therefore, providing a learning situation in which the learner can be reasonably self-reliant, which is important if they are to be fully supported by the program to practice as much as possible without the continual need for teacher supervision.

The involvement of SEN teachers is essential at all stages of a study of this kind. Their experience is invaluable at the storyboarding stage of a program design, where they are able to identify words, instructions, task actions that could be meaningless to a learner with low numeracy, and suggest ideas they know work, which saves time on developmental testing of the program itself. They are essential for administration of the programs at the appropriate stage in the curriculum. Once programs are reasonably well developed, and made available on the website, they offer further comments for improvement, and extension. Establishing an active online community of SEN teachers requires considerable management, however, as it is not yet a natural part of the professional teacher's work. For this to be a significant activity, it would have to be funded through further research.

5 Intervention study

The current intervention study[1] reported on here has developed interactive, adaptive programs on basic numerosity tasks: dot pattern recognition, matching collections of dot patterns to collections of digits (cards face up, and also face down, to rehearse memory for the patterns), a competitive game to match dot patterns and digits, a task to 'navigate' the number line to locate a target number, and a number bonds game, all available to download (http://www.low-numeracy.ning.com).

The design of the tasks uses the neuroscience to select the activity to be rehearsed, e.g. dot enumeration. The fact that these learners have atypical processing of numerosity means that the learning objective must be to make the concept of numerosity meaningful to them.

[1] 'Digital interventions for dyscalculia and low numeracy', funded by Becta small research grants.

Each program collects data on reaction times and the accuracy of learner responses. Appropriate customization is offered to teachers for each program (e.g. the range and type of numbers to be included). Each program, where possible, sets up a task that offers intrinsic feedback on the learner's action, to make the concept meaningful, and to enable them to see not just *that* it was wrong, but *why* it was wrong, and if appropriate, reconstruct the correct action. An example of the latter is the *Dots2Track* program, which helps learners recognize the canonical dot patterns as an aspect of the dot enumeration task. Learners in SEN groups can struggle for weeks to discern and remember these patterns, which are used as the basis for explaining further number concepts, such as bridging to tens, addition, subtraction and multiplication.

5.1 Dots2Track

The *Dots2Track* program is designed to help the learner discern the relationship between the numerosity in a dot pattern and its representation as a digit, and on a number line. It asks the learner to identify how many dots there are in a pattern, and for an incorrect answer, shows the pattern their input matches, counts the pattern onto a number line, and does the same for the target pattern, showing how the two compare (see Fig. 4). It then invites the learner to fix their own line by adding or subtracting one dot at a time. Once the regular patterns are known, the next level moves to random patterns, and uses the iteration rule to move the learner towards recognition, rather than counting to estimate the numerosity.

Each program is adaptive to the learner's needs, i.e. it selects the next task or trial according to the learner's performance so far. For example, in the *Dots2Track* program, it introduces new dot patterns only when some have already been mastered, according to the rules defined in the teacher preferences (how many times a pattern must be correctly identified). The program is, therefore, adaptive to the individual learner, to keep them always in the *zone of proximal development* as they develop their capability in dot enumeration.

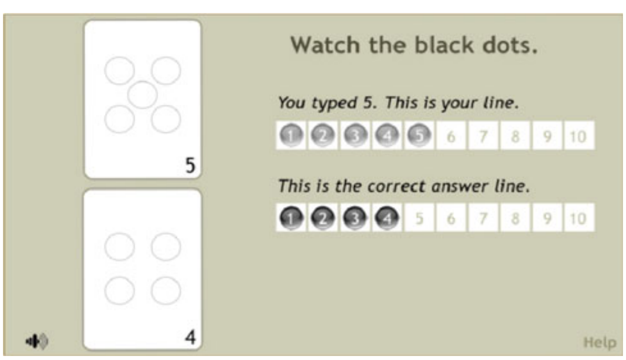

Fig. 4 The Dots2Track program showing the feedback when a learner types in '5' as an estimate for a pattern of 4. The 5 dot card appears, and the program counts those dots onto a numberline, enabling the learner to relate the outcome to their action. The 4 dots are then counted onto the line, so that at the next step the learner can click on either an 'Add one' or a 'Take one' button to construct their own version of the correct line

5.2 Dots2Digits

The *Dots2Digits* program is based on a standard task in SEN numeracy classes, where learners have to pair up a dot pattern with its appropriate digit, and vice versa. Teachers use many simple card games of this type, both for individuals and for pairs or groups of learners. For example, in one competitive version: dot cards are blue and digit cards are yellow; all the cards are kept face down; a learner turns up two cards and if they match keeps the pair and has another go; if they do not match they turn them over again; the next learner turns up a card which may be the pair of one already revealed, but they have to remember where it is to make the match. The value of the game is partly to rehearse the arbitrary relationship between patterns and their number names, but also to give the learners practice in memorizing the patterns. There is a strong incentive to try to memorize the pattern and its name. The digital version works in much the same way, storing the matched pairs visibly on the screen. An extension enabled by the technology is that at a higher level the cards can be displayed for a fixed (and reducing) time, to motivate the individual learner to work at memorizing the pattern. The digital version also adapts the task by beginning with a small number of pairs (say numerosities 1–4) and then shifts the range (to 2–5, up to 7–10) and then moves on to tasks with more pairs to match, up to 1–10.

6 Evaluation and findings

Evaluation of the programs is carried out through observation of the children using them, which enables improvements to be made to the interface to ensure that their attention is always productively focused on the cognitive task. Data on the means of errors and RTs can then be tracked across successive tasks for each learner. This was possible within a single session, but tracking individual learner performance across weeks does not produce reliable data, because of the multiple interfering variables due to their other number-related experiences in that time. Two individual cases illustrate the way progress, or not, can be demonstrated.

Figure 5 shows an example of the kind of within-task data that can be monitored for a Year 3 learner identified as dyscalculic.

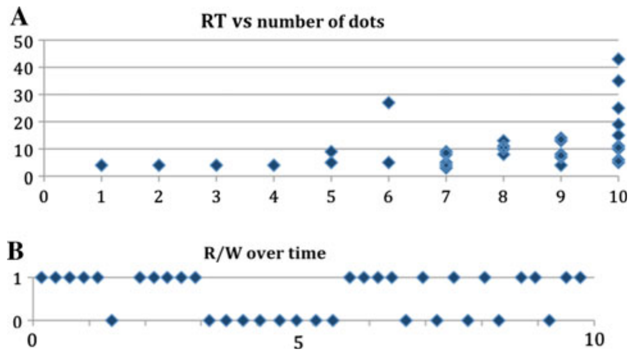

Fig. 5 Data from a Year 3 learner. **a** Shows that she takes an atypical amount of time to enumerate sets of 6 or more dots, and (not shown) she makes many errors in this range. **b** Shows her performance over 10 min. The first set are small numbers she knows and she gets them right (scoring 1), with few errors (scoring 0); the next numbers are larger and she makes errors. As she progresses through the program she makes more correct responses (scoring 1). **a** Reaction times for dot enumeration tasks, by number of dots. **b** Correct and incorrect responses on the trials over a 10-min period where 1 is correct and 0 is incorrect

Some learners continue to have difficulty in estimating the numerosity of dot arrays. Table 1 shows data from a Year 4 (9-year-old learner) working with *Dots2Track*. Here, the program has manipulated the exposure times of the dot arrays. In the first three sessions, with unlimited time, she is slow but accurate, insisting on counting the dots aloud. By session 2a, the program has reduced the display exposure to 1 s, requiring recognition of the pattern, rather than counting, and she makes far more errors; however, when the exposure is increased to 3 s, errors are reduced and speed improves.

By adjusting the exposure time in a systematic way, the program can adaptively assist the learner in progressing from counting to recognizing the number of elements in a visual array, emulating the practice of the SEN teacher in the classroom.

The adaptivity of the Dots2Track program means that new numbers are introduced when the learner is accurate and quick on the current numbers. Figure 6 shows four Year 4 low numeracy learners and their RTs. When each new number is introduced, the learner responds much more slowly, but responses speed up as they become more practiced, giving a characteristic saw-tooth pattern.

6.1 Time on task

We have compared the number of trials for each *Dots2Digits* task achieved by learners using digital interventions with learners doing the same task in special needs classes.

Here, we found that 8 SEN children (8 years old) playing the dots to digits game completed between 4 and 15 separate trials per minute, with an average of 8, whereas 2 of them set the same task in a class with a specialist teacher, using physical cards, managed only 2 trials per minute. This was partly because, as the children were essentially unsupervised while the teacher dealt with another child, they often argued about the rules of the game; they also tended to fumble the cards. Neither problem occurs in the digital game. Similarly, for six older SEN learners (12-year-olds) using the number bonds game, 4–11 trials per minute were completed, while in an SEN class of three supervised learners only 1.4 trials per minute were completed during a 10-min observation, partly due to off-task elements. Attention tends to be more focused in the digital game environment.

These data show that the learner can get much more practice with a digital game than is usually possible in a class with a teacher. Using the game therefore can free the teacher for one-to-one work while enabling learners to do independent unsupervised but computer-supported practice. There is emerging evidence for some learners that practice improves performance within the space of a lesson; improvements over time (weeks and months) are also identifiable when learners return to the same game, but these cannot be identified with the effects of these games alone.

6.2 Teachers' evaluation

One crucial source for evaluating the effectiveness of an intervention is how the teachers evaluate it. We have two ways of collecting these evaluations: first, by talking with teachers using the intervention in school, and second, through comments by the teachers in the online forum at http://www.low-numeracy.ning.com. They have been enthusiastic, so far. Here are some of the things they have said about the interventions:

Table 1 Change in the performance on Dots2Track of a Year 4 learner over successive stages of the adaptive program

	Session 1a	Session 1b	Session 1c	Session 2a (1 s)	Session 2b (3 s)
Errors	1	0	2	5	2
Mean RT	4.9	4.3	3.8	4.4	3.8

Sessions 1 and 2 are 3 weeks apart

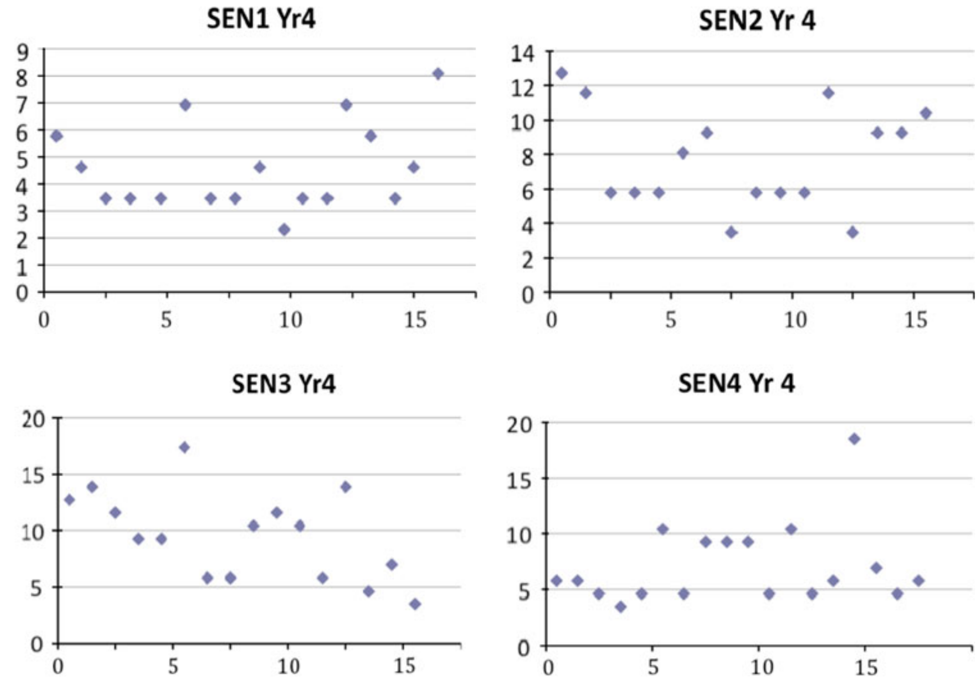

Fig. 6 Dots2Track results for four 10-year-old children with dyscalculia. The *horizontal axis* represents the number of trials to get all ten numbers correct and the *vertical axis* represents the time taken for each trial. When performance reaches criterion in speed and accuracy, a new number is introduced resulting in increased reaction time. This creates the characteristic saw-tooth pattern. The mean RT for typical 10-year-olds is under 2 s

- "Wow! It is so amazing to see all this work that links so perfectly to everything that I teach! I want to use the programmes straight away with the boys! They would be very useful in backing up all the concrete work we do and also in supporting independent work. I have often wished there were programmes like this! Thank you for sharing them :)" (Teacher on website)
- "These are exactly the kinds of programs my learners need" (SEN teacher)
- "The nursery children this year are much more number aware after playing with these games" (Primary teacher)

In addition, because we test storyboards and initial prototypes with teachers there are numerous comments on layout, wording, and sequencing of tasks that result in frequent changes to the design as it develops. We must stress that we are still at the stage of formative evaluation of these digital methods, as well as formative evaluation of the progress of the learners.

7 Conclusions

The starting point for our research has been behaviour: able children unable to learn arithmetic. This phenomenon has been a puzzle for many teachers and many researchers. A variety of different explanations, both expert and lay, have been offered for why this happens, and doubtless most of them can be documented in some poor learners.

For example, in an influential paper, Geary (1993) argues that dyscalculic "children show two basic functional, or phenotypic, numerical deficits":

1. "The use of developmentally immature arithmetical procedures and a high frequency of procedural errors" (p. 346)
2. "Difficulty in the representation and retrieval of arithmetic facts from long-term semantic memory" (p. 346)

Here, the idea is that there is a developmental progression from calculation strategies, such as counting on, to established associations between problems and their solutions. "Mastery of elementary arithmetic is achieved when all basic facts can be retrieved from long-term memory without error… [which in turn] appears to facilitate the acquisition of more complex mathematical skills" (Geary 1993, p. 347). According to Geary, laying down these associations in long-term memory depends on maintaining the problem elements (for example, two addends, intermediate results, and solution) in working memory. In addition, the use of immature or inefficient calculation strategies will risk decay of crucial information in working memory. However, although there is some evidence correlating measures of span with mathematical performance, one study found no difference on a non-numerical task testing phonological working memory (non-word repetition), suggesting that dyscalculic children do not have reduced phonological working memory capacity in general, although they may have a specific difficulty with working memory for numerical information (McLean & Hitch, 1999). Moreover, Landerl et al. (2004) found dyscalculia even when matched against controls with comparable spans. In this study, dyscalculics (who were in the bottom 2% of their age group on timed arithmetic) were also matched on IQ. This suggests that general cognitive ability alone is not a sufficient explanation. There is abundant evidence now that it is possible to be excellent at arithmetical calculation with low general IQ (see, e.g., Butterworth, 2006)

Another idea appealing to both lay and expert opinion is that failures to learn arithmetic can be due to poor linguistic skills. There are intuitive grounds for this. First, most formal arithmetical learning comes through language; second, people speak of a "language of mathematics"; third, it has been argued that arithmetical facts are stored in a verbal form, so that impaired language could affect the storage of useful facts; fourth, there is a statistically significant comorbidity between arithmetical disabilities and dyslexia. Dyslexia is usually a deficit in language abilities that affects phonological processing which is known to reduce working memory capacity (Nation, Adams, Bowyer-Crane, and Snowling, 1999) which in turn may affect lexical learning as well (Gathercole, 1995). These considerations imply that dyslexics should have difficulty with fact retrieval, if these are stored in verbal form, and with multidigit arithmetic with high working memory load. The problem with this line of argument, as discussed above, is that, as we have seen, dyscalculics do not have reduced working memory span. Moreover, both Shalev, Manor, & Gross-Tsur (1997) and Landerl et al. (2004) found no quantitative differences on tests of arithmetic, or on simple number tasks such as dot enumeration and magnitude comparison. Even in severe language disabilities, such as specific language impairment, there appears to be no effect on the basic capacities described here (Donlan, Bishop, & Hitch, 1998), though it may affect learning arithmetic in school (Cowan, Donlan, Newton, & Lloyd, 2005).

What is clear from recent research is that very basic domain-specific *core deficits* can have a severe effect on the capacity to learn arithmetic. Perhaps the best index of this is the efficiency of enumerating small sets of objects (typically dot arrays), although other indices have been suggested, such as comparing dot arrays (Halberda et al., 2008; Price et al., 2007). Our approach has been to use pedagogical principles and best teaching practice to motivate adaptive digital interventions designed to strengthen these capacities.

Our research programme is part of a movement to bring neuroscience, psychology and intervention closer together. Methods for teachers of dyscalculic learners based on the idea of a core deficit have already been developed (e.g. Butterworth & Yeo, 2004). Indeed, a pioneering adaptive digital method for strengthening the sense of number, called the *Number Race*, has been available for 4 years in pioneering research by Wilson, Dehaene et al. (2006) and Wilson, Revkin, Cohen, Cohen, and Dehaene (2006). The *Number Race* game is based on a rather different pedagogical approach from ours, and may be more suitable for some learners.

Neither Wilson's program, nor ours has been subjected to formal summative evaluation. Clearly, this is a limitation to any recommendation of how to proceed. However, formative evaluation is in progress, and teachers and researchers can contribute to the process through the website.

In the near future, we would expect to use neuroimaging to be part of summative evaluation of intervention for dyscalculia, in the way that it has been used already to assess the effects of phonological intervention on reading development in dyslexics. There it has revealed that the intervention tended to align neural activity with the patterns found in typical readers, suggesting that the reading process was becoming more typical rather than just more accurate using atypical compensatory strategies (Eden et al., 2004), a comparable approach to evaluate dyscalculia interventions is envisaged.

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