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The implications for education of an innate numerosity-processing mechanism

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One specific cause of low numeracy is a deficit in a mechanism for representing and processing numerosities that humans inherited and which is putatively shared with many other species. This deficit is evident at each of the four levels of explanation in the 'causal modelling' framework of Morton and Frith (Morton and Frith 1995 In *Manual of developmental psychopathology*, vol. 1 (eds D Cichetti, D Cohen), pp. 357–390). Very low numeracy can occur in cognitively able individuals with normal access to good education: it is linked to an easily measured deficit in basic numerosity processing; it has a distinctive neural signature; and twin studies suggest specific heritability, though the relevant genes have not yet been identified. Unfortunately, educators and policymakers seem largely unaware of this cause, but appropriate interventions could alleviate the suffering and handicap of those with low numeracy, and would be a major benefit to society.

This article is part of a discussion meeting issue 'The origins of numerical abilities'.

1. Introduction

Napoleon famously said that mathematics is 'intimately connected with the prosperity of the state' (cited by Boyer & Merzbach [1], p. 466). In his foreword to the Cockcroft 1982 report on mathematics education, Sir Keith Joseph, Secretary of State for Education and Science, wrote 'Few subjects are as important to the future of the nation as mathematics' [2, p. iii]. Since Cockcroft, in the UK alone there has been Professor Adrian Smith's report on post-14 maths [3] and Sir Peter Williams' report on primary maths [4]. Similarly, the US National Research Council [5] noted that 'The new demands of international competition in the twenty-first century require a workforce that is competent in and comfortable with mathematics'; and to that end 'The committee [of experts] was charged with examining existing research in order to develop appropriate mathematics learning objectives for preschool children; providing evidence-based insights related to curriculum, instruction and teacher education for achieving these learning objectives' (p. 1). This concern for the general level of mathematics—I will return specifically to numeracy in a moment—is understandable, and in economic terms, justifiable. An OECD modelling exercise showed that the level of maths attainment is a causative factor in long-term economic growth.

These reports focused on the standards of teaching and teachers, so for example, Cockcroft required better mathematics education for teachers, and Smith's report recommended the setting up of a National Centre for Excellence in the Teaching of Mathematics (which indeed happened). Improving teaching would, it was implied, improve the average level of attainment. More specifically, a study by the UK's Department for Education and Employment in 1998, titled *Numeracy Matters*, made a range of recommendations about the teaching of numeracy to raise the average level of attainment [6].

Now a national average does not show the range of individual differences in maths attainment. According to the most recent Programme for International Student Assessment (PISA) study, '23% of students in OECD countries, and 32% of students in all participating countries and economies, did not reach

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the baseline Level 2 in the PISA mathematics assessment of 15 year olds. At that level, students can only extract relevant information from a single source and can use basic algorithms, formulae, procedures or conventions to solve problems involving whole numbers' [7, p. 4]. This failure to reach Level 2 varied from 3.8% in Shanghai, China to 74.6% in Peru. In the USA, it is 25.8% and in the UK it is 21.8%.

These reports did not address the question as to why some learners are stuck at the bottom of the distribution, apart from inadequate access to appropriate teaching. The recently created charity, National Numeracy, aims to raise the standard of numeracy in the UK. Their starting point is 'that a major shift in attitudes is key to this'. The problem lies with 'negative attitudes' (https://www.nationalnumeracy.org.uk).

Of course, there have been an enormous variety of studies using a variety of social and cognitive independent variables to account for a wide variety of dependent numeracy measures. There is no space in this brief review even to list them all. Among the social factors are parental education, socio-economic status and teaching quality [8]. The drivers are intercorrelated to some extent, and they are linked by the amount of numerical activities in the home. That is, parents, especially mothers, with higher educational levels are more likely to carry out numerical activities with their children, including sharing counting practices, explicitly naming numbers when shopping, counting steps and body parts, and so on. These activities, though associated with parental education and socio-economic status, have a specific effect over and above these two variables [9].

Among the cognitive factors most usually cited are intelligence and memory. Studies have found a correlation between IQ scores and maths attainment in school. To take just two representative examples of such studies: a study in adults, of non-verbal intelligence measured by Ravens Advanced Progressive Matrices [10], spatial ability and processing speed-predicted SAT maths scores [11], and a study of the effects of IQ in first Grade pupils in the USA, found that low IQ children compared with their peers were significantly worse on a standard test of mathematical reasoning, and on number naming, writing numbers to dictation and on number comparison [12]. Low IQ was defined as $IQ < 85$ (1 s.d. below the population mean) and average to high IQ ($IQ > 85$) classified on the basis of vocabulary and block design (from the WIAT [13]). However, as I will show in detail, normal or superior IQ is not sufficient for good numeracy; and low IQ is not sufficient for poor numeracy. So, for example, Zacharias Dase (1824–1861), an extraordinary calculator who, for a time, assisted Gauss in calculating tables, was credited by distinguished mathematicians, including his collaborators, with 'extreme stupidity'. One pair of twins with prodigious abilities for calendrical calculation were estimated to have IQs in the 60s [14]. Mitchell in a review of mathematical prodigies noted that two prodigious calculators, Fuller (1710?–1790) and Buxton (1702–1772), 'were men of such limited intelligence that they could comprehend scarcely anything, either theoretical or practical, more complex than counting' (pp. 98–99). One autistic boy, unable to speak or understand language, and with a very low measured IQ, was able to identify large primes and to extract factors of other numbers faster than an adult with a maths degree [15].

It is a characteristic of low-attaining children to have trouble recalling arithmetical facts, and the worse they are, the lower proportion of their *correct* addition answers are solved by retrieval [16]. So why do children with low or very low attainment fail to use retrieval? Is there something wrong with long-term memory in general, either in storage or retrieval processes? Unfortunately, this is rarely if ever tested in low attainers except with arithmetical material.

Short-term, or 'immediate', memory span—immediate repetition of words or non-words—has been correlated with addition accuracy and strategy sophistication in 6-year-olds [17]. Studies specifically of mathematically low-attaining children found effects only when related to memorizing numerical material, such as forward digit span [18,19]. In high-attaining calculators, such as Rüdiger Gamm, we find domain-specific effects. When formally tested by Pesenti *et al.* [20], Gamm had a forward span of 11 digits (controls 7.2, s.d. = 0.8) and 12 digits backwards (controls 5.8, s.d. = 0.8), whereas his letter span was in the normal range.

Working memory, as characterized by Baddeley and Hitch, is a more complex construct that includes a Central Executive that updates information to make it relevant to the current task [21,22]. Most studies that have looked at both span and updating have found that it is the updating function that is most closely linked with arithmetical ability. One study found that high and low achievers who did *not* differ on forward digit span did differ on backward digit span, an index of the efficiency of the Central Executive component of working memory because the item order needs to be updated [23]. Again, there is evidence for domain specificity in working memory: we devised a novel updating task in order to see whether the low attainers were worse at updating numerical material only (domain-specific) or on non-numerical material (domain-general). Participants were asked to recall the smallest item in a spoken list. The list could consist of numbers, e.g. '26–68–92–66–35' or animal names, e.g. 'giraffe–pelican–tortoise–tiger–chicken–dolphin'. This experimental design could be manipulated so as to increase the number of items to be recalled (*load*)—that is, the smallest item, or the two, or the three smallest items—and also the number of items to be inhibited (*inhibition*). Overall, the low-attaining group did better on the animal task, while the typically attaining group did better on the number task. Neither load nor inhibition distinguished the two groups, except in one specific way: when more items had to be inhibited, this had a greater effect on animal recall than number recall in the typical group, while it had a greater impact on number recall than animal recall in the low-attaining group. This suggests that working memory could be at least partially domain-specific [24]. It also supported the hypothesis that it was not the ability to hold information—as measured by span—but rather to modify it in light of current task demands that was important.

The official reports neglect the possibility that, along with domain-general cognitive tools needed for learning anything, there is a domain-specific tool in the *starter kit* for learning to become numerate. The cultural environment provides (or sometimes fails to provide) symbolic resources to make numeracy more efficient: counting words, numerals, body-part representations and other external representations, such as tally ticks and marks on bones and stones (see the paper by d'Errico *et al.* in this issue [25]) [26]. It will turn out that even for learning to understand these symbolic resources both domain-specific and domain-general tools are needed.

I call this domain-specific tool the *numerosity tool*. In typical learners, the numerosity tool, I will argue, supports the normal development of arithmetical competence. (I have called it the 'number module' elsewhere [26].) When this tool is inefficient, normal development is seriously handicapped. I will review some of the studies we have carried out to test this idea. To make this case, I organize the evidence along the lines proposed by Morton & Frith [27] in their 'causal modelling' framework, to show domain-specific individual differences in behaviour, in cognition, in the brain and in genetics.

2. The behavioural level: selective disabilities in arithmetic

I start by describing briefly the survey evidence for individual differences in arithmetical competence, and then go on to show experimental evidence of selective disabilities in arithmetical competence.

A large-scale longitudinal UK survey of numeracy, based on a representative sample of about ten thousand people, found that around 20% of 20- and 34-year-old adults tested are at 'entry level 2'—the level expected of 9-year-olds [28], and of these only about a quarter report having difficulties with numbers or arithmetic [29].

The UK cohort study showed that although 14% of individuals have both numeracy and literacy difficulties (below Entry Level 2), 11% have poor numeracy alone. This suggests that numeracy problems can be selective. Indeed, we have found that many high-functioning individuals have selective deficits in their numerical abilities. Here are some examples of their self-reports.

- Paul Moorcraft, defence correspondent, novelist,
  - '[Maths] was like being asked to speak in an unknown foreign language'.
- Emma King, cosmologist
  - 'I can't add up, subtract or multiply in my head ... I calculate $4 + 3$ by counting'.
- Vivienne Parry, broadcaster, science journalist
  - 'I was in the top set for all other subjects ... No matter how hard I tried or how much homework I did, I just didn't get it'.
- Articulate 8-year-old, good at all other school subjects, expert on dinosaurs, but 'the only subject I don't like is maths'
  - Teacher: *What do you need to add to 8 to make 30?*
  - Child: *Two* (Even with the help of Cuisenaire rods and a UTH board, he fails to work it out.)
- BD, a 23-year-old reading English at an Ivy League university
  - Experimenter: *Can you please tell me the result of nine times four?*
  - BD: *Yes, well, looks difficult. Now, I am very uncertain between fifty-two and forty-five... I really cannot decide: it could be the first but could be the second as well.*
  - Experimenter: *Make a guess then.*
  - BD: *Okay... uhm... I'll say forty-seven.*
  - Experimenter: *Good, I'll write down forty-seven. But you can still change your answer, if you want. For example, how about changing it with thirty-six?*
  - BD: *Bah, no... it does not seem a better guess than forty-seven, does it? I'll keep forty-seven.*

(recorded interview by Rusconi, Losiewicz & Butterworth, cited in [30], pp. 67–68)

When adults with severe numerical disabilities of this kind are formally tested on a variety of cognitive tasks including non-numerical quantitative tasks, many show a deficit on specifically numerical tasks. In one study [31], severe numerical disability in the presence of normal or superior IQ (WAIS-R) was diagnosed on three criteria.

1. Below cut-off on standardized arithmetical tests: the Graded Difficulty Arithmetic Test [32] and the arithmetic subtest of WAIS-R [33].
2. Numerosity discrimination was significantly worse than controls; [34].
3. Met the criteria for *dyscalculia* on the capacity and attainment subscales of the dyscalculia screener [35].

These participants and matched controls were tested on two tasks of continuous quantity: one required discriminating the duration of two stimuli and the other required the discrimination of two line lengths. The numerical tasks required selecting the larger of two numbers; comparing a number with an array of dots; and verifying a simple calculation (figure 1). (We will see below in §4 the neuroimaging results from this task). The study in figure 1 is experimental evidence of a disability specific to numerical tasks and not tasks involving continuous quantity. Therefore, it is important to distinguish numerical abilities from quantitative abilities in general, which may or may not include numerical abilities.

The criteria listed above constitute a behavioural way of identifying a disability usually called 'developmental dyscalculia' (and sometimes, 'mathematical learning disability'). However, as we will see in the next section, the critical characterization of this condition is cognitive: a core deficit in the efficiency of the numerosity tool.

3. The cognitive level: a domain-specific tool in the starter kit for arithmetic

A selective deficit in numerical abilities, when the learner has good cognitive capacities and access to a normal education, suggests that there is another ingredient in the arithmetical *starter kit* that could be functioning inefficiently.

Talks at the Royal Society meeting The Origins of Numerical Abilities, and represented in this volume, show that many other species possess numerical capacities, and not just those with big brains, such as monkeys (1–4 billion neurons) but also creatures with small brains, such as bees (approx. 1 m [36]), fish (approx. 10 m [37]) and anurans (approx. 12 million neurons). It is possible, and perhaps likely, that they share the same mechanism deployed to extract numerosity information from the environment as humans, albeit within distinct evolutionary and neural contexts.

If there is a number tool starter kit, what form might it take? One proposal about the nature of the mechanism that has been used to explain both human and non-human numerical abilities is the *linear accumulator*. For a recent mathematical account of a linear accumulator, see [38].

The linear accumulator has several attractive features as a key tool in the starter kit, along with the domain-general tools mentioned above. First, the mechanism is inherited, with the gate and storage mechanisms evolved to fit the objects of

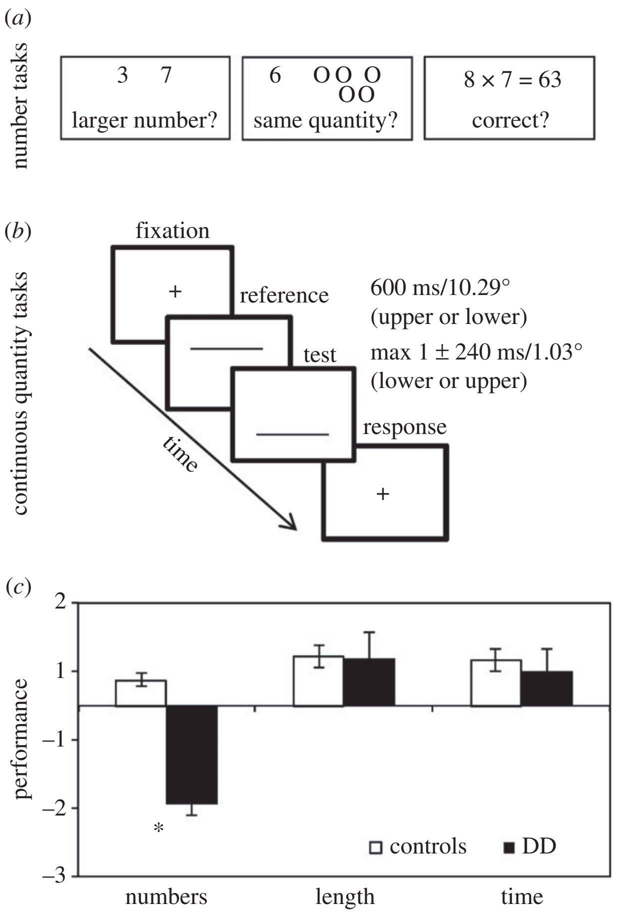

Figure 1. (*a*) Tests of numerical ability. (*b*) Test of abilities to discriminate continuous quantities, where one condition requires deciding whether the duration of the test is longer than the reference; in the other, whether the size of the test stimulus is longer than the reference. (*c*) The normalized results in standard deviation units. Adults with severe numerical disabilities (DD) were significantly worse than controls only on the number tasks. From Cappelletti *et al*. [31].

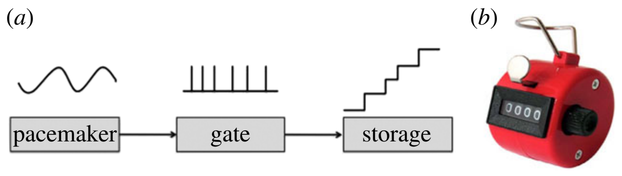

Figure 2. (*a*) A sketch of a neural accumulator mechanism: the brain contains a 'pacemaker' that generates quanta of energy and a 'gate' that lets through a quantum for each object to be enumerated. The quanta are stored in the 'storage' linearly proportional to the number of objects enumerated. Adapted from Gibbon *et al*. [40] and Meck & Church [41]. (*b*) Tally counter. (Online version in colour.)

interest for that species, so it does not need to be learned. Of course, other processes will need to be learned: for example, how the accumulator 'level' is linked to counting words [39] in a way that maintains the link between numerosity (cardinality) and order (ordinality). Second, it can be a very simple device like the tally counter in figure 2, which can be used to count the number of sheep but not goats in a field. The counting just depends on a button press; the more complicated cognitive process is deciding what is a sheep when they are all moving about and there are both young and old of each species. This will require knowledge of the species' characteristics, among other cognitive capacities. Third, being a linear accumulator, arithmetical operations are carried out in a natural way, whereas addition and subtraction would be difficult in a logarithmic accumulator [39]. Fourth, there is evidence that single neurons can implement an accumulator [42], which could explain why even animals with very small brains can carry out the simple numerical processes that would be supported by an accumulator. See §4 below for more on neural processes. By contrast, evidence collected so far suggests that other animals are much more limited in the presentation and modality of objects they can enumerate. For instance, some species of ants can count their own steps in order to estimate distance from their nest [43], but it is not clear whether they can count anything else. Bees can count landmarks, and also small arrays of objects, but again this seems to be the limit of the counting ability (see [44] for a review).

It should be noted that computational models of numerical cognition include an accumulator stage (sometimes called a 'summation' stage) in which inputs are normalized so that each item to be enumerated is represented in the same way—Difference of Gaussians filters in figure 3*a*, arbitrary units in 3*b* and 3*c* [45–48].

Now, if the accumulator is the key *numerosity tool* in the starter kit for acquiring arithmetical competence, then individual differences in its efficiency will be linked to measures of arithmetical performance. No test will be a pure measure of the efficiency of the accumulator. Consider a test that compares the numerosities of two sets, such as those in figure 4*a–c*. In these cases, the total number of objects in the sets may induce different mechanisms: an attention-demanding enumeration for numerosities up to 4 [52,53], perhaps a sequential enumeration process if accuracy is a task demand from 4 to 8 or 9 [54] and a statistical extraction process for larger numerosities that does not depend on a linear accumulator [55]. On top of that, there will be a decision process that will also contribute to the test outcome (figure 3).

Another method commonly used for measuring individual differences in the efficiency of the numerosity tool is a dot enumeration (DE) task (see figure 5). Here, the participant simply names the number of objects in the array. Again the same questions about the enumeration processes can be asked, and, in addition, there is the issue of knowing, understanding and producing the name of the numerosity, or selecting the digit symbol for it. (One method that has been used extensively in animal studies, classically by Otto Koehler with corvids [57], but, as far as I know, never with humans, is a match-to-sample task. This task has the advantage of requiring the identification of a particular numerosity but without the requirement of knowing the meaning of number words or of numerals. The closest approximation to it is a study of Australian Aboriginal children, whose languages contained no counting words, and whose culture had no counting practices. The children were required to construct from memory the numerosity of a sample array and could not therefore rely on remembering a symbol standing for the numerosity [58].)

Despite these issues, these relatively simple metrics correlate quite well with arithmetical competence, suggesting a role for the numerosity tool in the development of arithmetic. Thus, Halberda *et al.* found a significant association between a numerosity comparison task (figure 4*a*) in the ninth grade

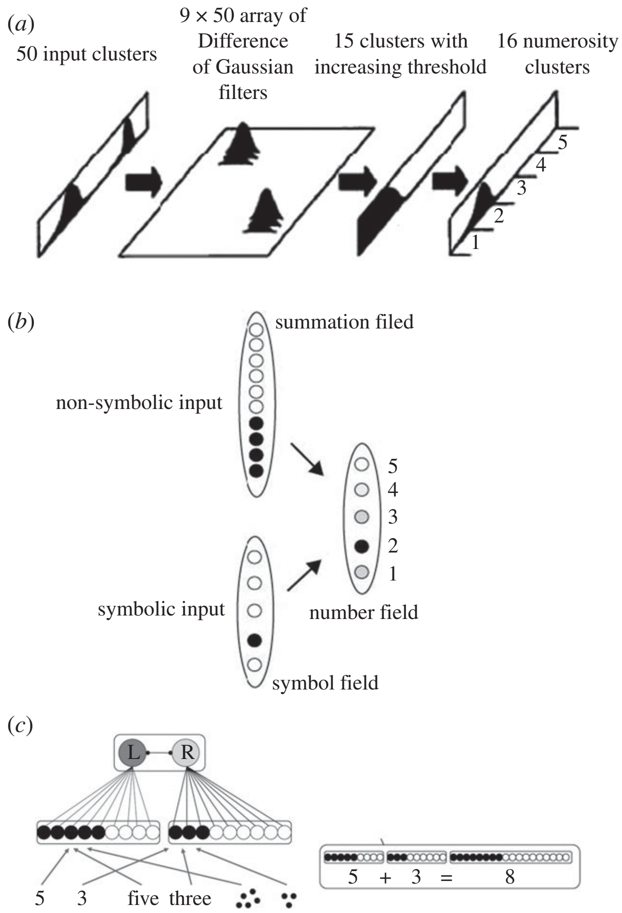

Figure 3. Neural network models of simple numerical cognition containing an accumulator. (*a*) The accumulator is called a 'cluster' that accumulates linearly, from Dehaene & Changeux [45] (*b*) A similar model that includes symbolic as well as non-symbolic—sets of objects—as input. The accumulator stage is called the summation field [46]. (*c*) A model that includes several accumulators: in this example, two that have discrete numerosity coding that represents the semantics of both symbolic and non-symbolic input in the same way (in contrast to (*b*)) and one that is used to combine the representations in a linear addition process [47–49].

and a standard arithmetic test in kindergarten, with association becoming stronger for older children (R^2s between 0.127 and 0.326 depending largely on age) [34]. Piazza *et al.* similarly found a weaker correlation between numerosity comparison (figure 4*b*) and some but not all measures of arithmetical ability [50]. In a recent meta-analysis of 41 studies, an average correlation between these measures and measures of mathematical competence was $r = 0.241$ but with only a weak effect of age, and several studies found no correlation [59].

Stronger evidence for the role of a numerosity tool in arithmetical development is a longitudinal study of 159 children by Reeve *et al.* [56] that measured efficiency in kindergarten (ages 5–6 years) and at four other ages using timed DE, and rather than using arbitrary criteria to assign learners to groups, a cluster analytic approach was adopted. Children were retested on DE at 7, 8.5, 9 and 11 years, and there was a significant ordinal correlation among clusters: that is, children tended to stay in the same cluster—fast, medium or slow—throughout the testing period. Cluster membership in kindergarten predicted age-appropriate arithmetical competence at several subsequent ages to 10 years, and key to our hypothesis is that membership of the slow cluster (7% of the sample) were always worse on age-appropriate arithmetic tests than the other two clusters.

Low efficiency in kindergarten is thus a good predictor of children who will have trouble with learning arithmetic in school.

Cluster membership was not significantly associated with measures of domain-general capacities: the simple reaction time (RT) measure at any age, even though DE efficiency is an RT measure (median F value $= 2.14$, $p > 0.1$). Nor was it associated with a measure of non-verbal intelligence (Raven's coloured progressive matrices [60]) ($F_{2,156} = 0.61$, n.s.). Nor, indeed, was it associated with the ability to access symbols, as measured by naming digits ($F_{2,156} = 0.66$, n.s.) or naming letters ($F_{2,156} = 0.15$, n.s.).

These findings support the hypothesis that an efficient domain-specific tool is needed for the typical development of arithmetic.

These results are consistent with the first study we carried out on dyscalculic 9-year-olds who had severe difficulties in accuracy and speed in timed arithmetic (3 s.d. below the sample mean) [61]. In fact, it was easy to match the dyscalculic children to controls in the same classrooms on tests of general cognitive ability (Raven's coloured progressive matrices [60]), plus WISC III Mazes subtest, and on forward and backward digit span [62], suggesting that dyscalculia is not a result of poor general cognitive capacity.

If an efficient numerosity tool is necessary for normal arithmetical development, is an inefficient tool *sufficient* for disabilities in arithmetical development? Evidence for this comes from a study of one district of Havana, Cuba—Havana Centro. It started with a sample of 11 562 children aged from 6.4–17.3 years—effectively every child in this district, all of whom underwent a curriculum-based group-administered standardized test each school year—see 'MAT' in figure 6, which also summarizes the sampling method and results [63].

Of these, 1966 were tested individually using a specially designed computerized 'basic numerical battery' (BNB) comprising

— Item-timed arithmetic (log EM)—the measure of arithmetical competence.
— Item-timed dot enumeration (log EM)—the measure of numerical capacity.
— Basic reaction time.

The BNB used an efficiency measure that combined accuracy and speed (median RT adjusted for basic RT/accuracy) to give LogEM.[1]

Of the 1966 tested, 9.4% were classified as *Arithmetic Dysfluent* (AD), meaning that they were 2 s.d. worse on timed arithmetic; 4.5% were classified as having a *Core Deficit* based on their performance on DE; and 3.4% were classified as *Developmental Dyscalculic* (DD) if they were both AD and had a core deficit.

Is a core deficit *sufficient* for AD? In terms of the *sensitivity* of the core-deficit test, the true-positive rate is 27%; that is, there are many causes of AD besides the core deficit. In terms of *specificity* (true negative (1570)/true negative + false positive (35)), the true-negative rate is 98%, positive predictive value 74% and negative predictive value 86%. That is, having a core deficit makes it almost certain that a child will have difficulty in being normally fluent in arithmetic. Incidentally, the standardized school test, MAT, is actually a

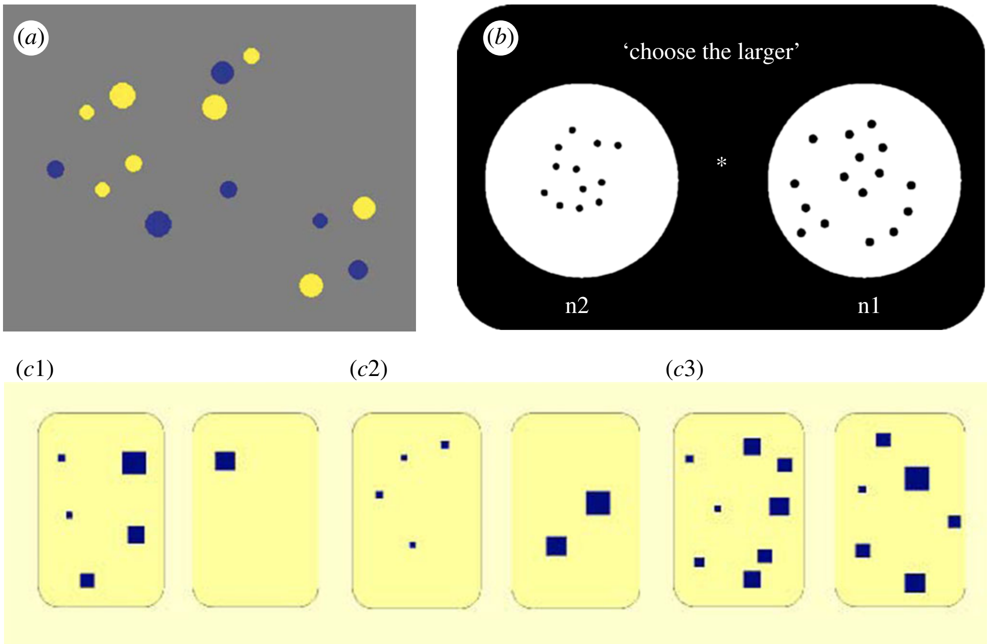

Figure 4. Three ways of testing the ability to discriminate numerosities while controlling for non-numerical dimension of the area of the objects. (*a*) The participant selects the colour of dots, blue or yellow, with the greater numerosity [34]. (*b*) The participant chooses the panel with the greater numerosity [50]. (*c*) In this method, numerosity and area are varied orthogonally, and the participant has to select the panel with the greater numerosity [51].

bad predictor of which child is going to have trouble and which is not (specificity 45%).

4. The neural level: neural basis of the 'core deficit'

This section will present evidence from studies of brain structure and function that reveals a specific network for numerical processes, and also reveals how this is abnormal in dyscalculics.

An accumulator mechanism (figure 2) can be very simple—analogous to a tally counter—and may need very few neurons to implement. In fact, there is some evidence for single neurons performing this task in monkey lateral intraparietal sulcus [42]. Even if the counter is neurally simple, identifying the objects to be counted may not be. Big brains are needed to decide what is an object to be enumerated, as in the sheep and goats example above.

Moreover, humans are flexible and can enumerate any set of denumerable objects, whether they are presented simultaneously or sequentially, in any modality and by a common neural mechanism ([64,65]; see also [66–68]).

There is now very extensive evidence that numerical tasks are supported by a specific brain network involving the left and right intraparietal sulci (IPS), with the left IPS more strongly linked to the frontal lobes (see meta-analyses of the neuroimaging evidence in [69–71] and reviews of the neuropsychological evidence in [26,72,73]).

Now these studies did not always distinguish between general quantification and specific numerosity processing. One study set out to distinguish numerosity that is discrete from continuous quantity in non-symbolic tasks (figure 7), and more specifically tested the idea that there was a common neural mechanism for enumerating collections of objects presented simultaneously and those presented sequentially [64].

First, we asked whether collections of discrete objects activated distinct neural regions when compared with equivalent continuous quantities. This they did, notably in the occipital cortex, but this is scarcely surprising because the visual stimuli are different. However, they also activated the parietal lobes distinctively: there was a small common region in both left and right IPS that was activated in both simultaneous and sequential presentations and, moreover, the amount of activation was parametrically modulated by the ratio of the two collections being compared.

Thus, for example, these regions were more activated by comparing 11 blue to 9 green than by comparing 14 blue to 6 green (as in figure 7). In this respect, the IPS activations were different from the occipital activations, which were not modulated by the ratio between collections, thus showing that although visual regions were sensitive to the differences between continuous and discrete stimuli, they were not sensitive to their numerical properties. The IPS activations were not only sensitive to these differences but also to their numerical properties. This is another reason why behavioural tests using dots in comparison, enumeration or matching to the sample are so important.

In the study by Cappelletti *et al.* [31], as described above in §2, the brain region most strongly correlated with and specific to the number tasks was the right intraparietal sulcus, while the region activated by both number and continuous quantity tasks was the right temporo-parietal junction (figure 8). This again supports the contention that the processing of numerosities is distinct from the processing of continuous quantities.

In children in the range 7–8 years, grey matter volume correlates positively with performance on a standardized test of arithmetic [74]. There is evidence that the grey matter volume of the IPS is lower in individuals with selective disabilities in numerical tasks. This was first demonstrated in adolescents with otherwise normal cognitive functioning when compared with matched controls [75]. This

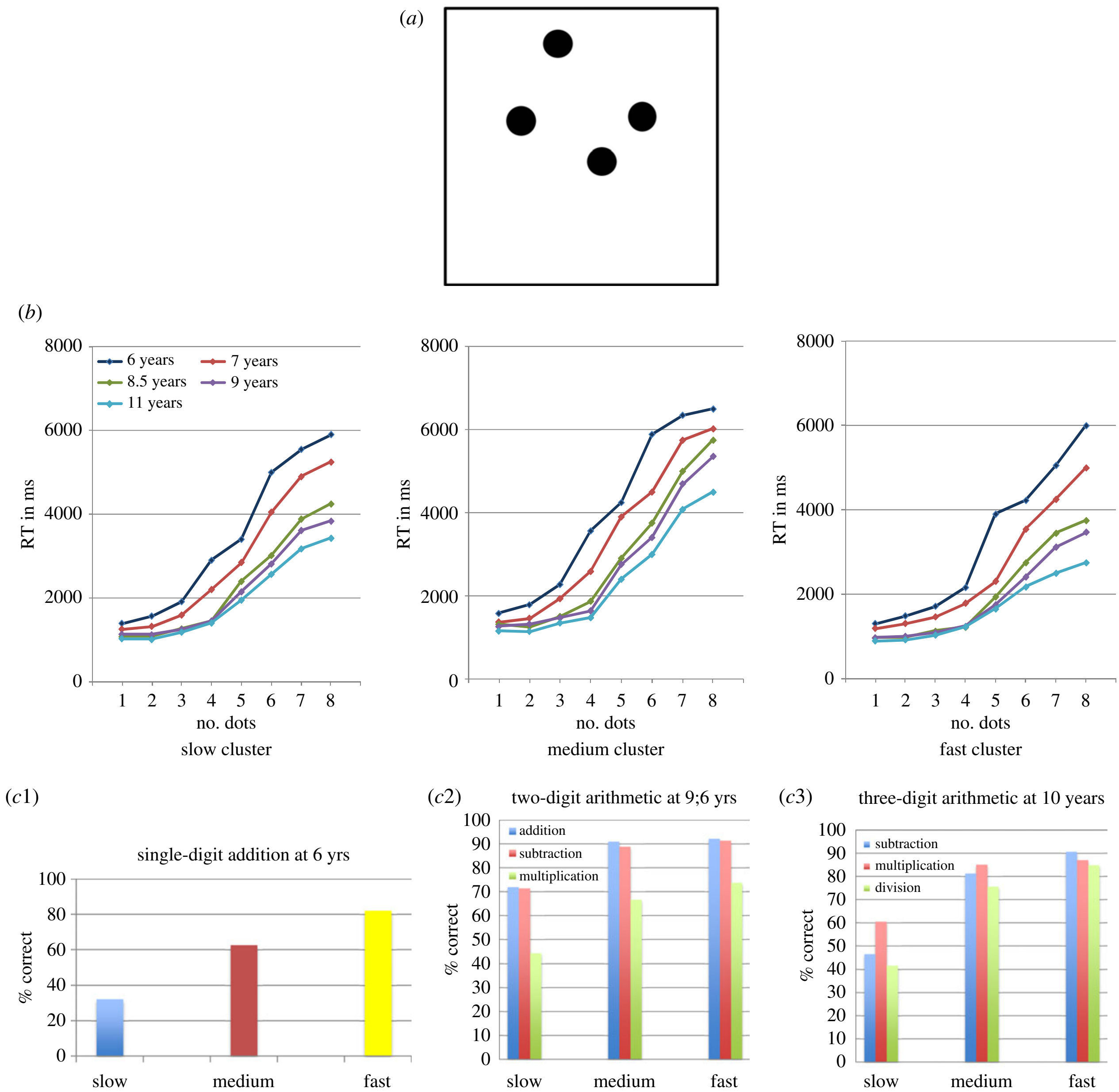

Figure 5. Melbourne longitudinal study of 159 children from kindergarten to age 11 years. (*a*) The dot enumeration (DE) task. Task: How many dots are there? Answer as quickly as possible. (*b*) The cluster structure in kindergarten, with the arbitrary names slow, medium and fast. (*c*) Age-appropriate arithmetical competence at three time points. At each time point the slow cluster defined at kindergarten (7% of the sample) is significantly worse than the two other clusters (adapted from Reeve *et al.* [56]).

study revealed reduced grey matter in the left IPS in adolescents with very low scores on a standardized test of simple arithmetic [75]. There is also some evidence that grey matter volume in relevant regions *increases* in typically developing 8- to 14-year-olds, but does not increase or even decreases in some dyscalculics [76], suggesting that dyscalculic brains are not just different but develop on a different trajectory.

Twelve-year-old children show abnormal and *reduced* activation in the right IPS during a magnitude comparison task similar to the one in figure 4*c* [77]. By contrast, a recent study of 7- to 9-year-olds carrying out single-digit addition verification (e.g. $4 + 3 = 8$?) found *overactivation* in several cortical areas including the bilateral IPS, but also in the prefrontal cortex [78]. The authors of this study note that 'The profile of differences in brain activation is currently an unresolved issue, as previous research has reported both over and under activation in MLD [mathematical learning disabilities], relative to control groups. This lack of consensus is likely due to the limited number of studies, the use of different cutoff criteria for identifying MLD groups . . . as well as the use of wide age ranges, and diverse experimental tasks and control conditions.' (p. 2).

One possible explanation of the apparent conflict between these studies is that increased arithmetical proficiency requires lower levels of activation during calculation [79]. Now, the rate of development of proficiency in calculation will be slower in low attainers than typical attainers, thus low attainers may show higher activation than their age-matched controls precisely because they are less proficient. By contrast, a study by Price *et al.* [77] of older children does not involve calculation proficiency but rather taps the core system of numerosity processing. However, it is not clear whether the developmental trajectory of this task differs between the dyscalculics and the typical developers.

Differences in the way the arithmetic network is connected have also been found in children with low numeracy. For example, a study of 7- to 9-year-olds found

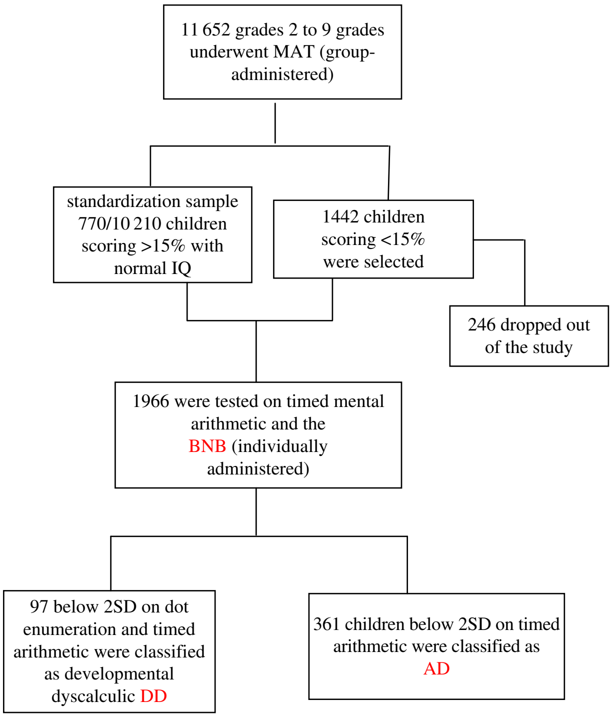

Figure 6. The structure of the Havana prevalence study [63] with a summary of the results. BNB, computerized basic numerical battery. (Online version in colour.)

that those with 'mathematical disabilities' (below 90 on the WIAT-II Numerical Operations subtest [13]), compared with controls, showed hyper-connectivity of the IPS with a 'bilateral fronto-parietal network', that is, the network known to be active when carrying out numerical tasks [80]. In fact, the resting state functional connectivity in links between regions known to be involved in calculation was negatively correlated with numerical operations. Now these children are not dyscalculic by the criteria I have been advocating. Indeed, some of them could be classified as normal, that is, within 1 s.d. of the population mean. It is not clear why the fronto-parietal network should be more connected than controls, except perhaps that lower calculation proficiency means that not only the relevant regions, but also the connections between them, are more active.

On the other hand, a study of 12 dyscalculics between 8 and 14 years relative to matched controls showed an age trend of *decreased* white matter volume, a structural correlate of connectivity, on some relevant tracts where the controls showed increasing white matter volume [76]. This suggests a contrasting hypothesis that one cause—or perhaps one consequence—of dyscalculia is a relative failure to link up the relevant brain regions adequately. (See [81] for a cognitive account of atypical links between numerosity processing (IPS) and number symbols (frontal) in dyscalculia.)

5. Genetics of the core deficit

A key question in mathematical cognition is to determine the extent to which mathematical abilities and disabilities are shaped by an individual's genetic endowment. That is, is there a genetic reason why some people are bad at maths and other people are not? Of course, there will be many reasons for individual differences, including experience with mathematics, and to establish whether there is a genetic reason, other possible reasons need to be factored out. In particular, is there a genetic basis for the core deficit?

There are two alternative genetic hypotheses.

(1) Numerical abilities are part of our general cognitive endowment, and are closely linked to the inheritance of other cognitive abilities. Therefore, there should be no genetic influences specific to numeracy.
(2) Numerical abilities are based on a specific genetic endowment.

There are grounds for the first hypothesis: first, that measures of mathematical abilities are correlated with measures of general intelligence; second, that measures of mathematical ability are correlated with performance on core school skills, such as reading. It has been argued that most, if not all, cognitive functions involve 'generalist genes'. 'In the "generalist genes" hypothesis, it is suggested that the same genes affect most cognitive abilities and disabilities. This recently proposed hypothesis is based on considerable multivariate genetic research showing that there is substantial genetic overlap between such broad areas of cognition as language, reading, mathematics and general cognitive ability' [82, p. 198].

At the same time, there are grounds for preferring the second hypothesis. First because some 30% of the genetic variation in a large twin sample of 7-year-olds is specific to mathematics performance in school [83]; see below. Second, there is the specificity of the neural substrate for core arithmetical abilities, and the selectivity of dyscalculia, as we have shown. We have noted that individuals from widely different cultures—from trading and non-trading cultures, and even from cultures with no counting practices and no counting words—nevertheless possessed a capacity for making decisions on the basis of numerical information [58].

We should also note that infants, even in the first week of life, before they had much opportunity to learn about numerosities, could make discriminations on the basis of the numerosity of visual displays [84,85], and that at six months they responded more abstractly, to the correlation of the numerosity of a visual and an auditory display [86]. At this age, they can also carry out very simple addition and subtraction [87]. This suggests that the bases of these capacities is not induced by education or culture but is innate.

(a) Gender

One important genetic factor is gender. Males and females differ systematically, of course, on their sex chromosomes; typically, females have two similar X chromosomes (XX) and males have an X and a much smaller Y (XY). This difference in the sex chromosomes has many consequences, not least differences in hormones.

Detailed and extensive studies of gender differences in the USA suggest that there is no significant gender differences for numbers and arithmetic [88], and the same factors—genetic, shared and non-shared environments—affect both males and females in the same way in both typical and low attainers [89]. For a numerosity discrimination task (figure 4*a*), where one would expect very little environmental effect, again, there appears to be no difference in the mean accuracy or variance between boys and girls, and again the same factors affect both [90].

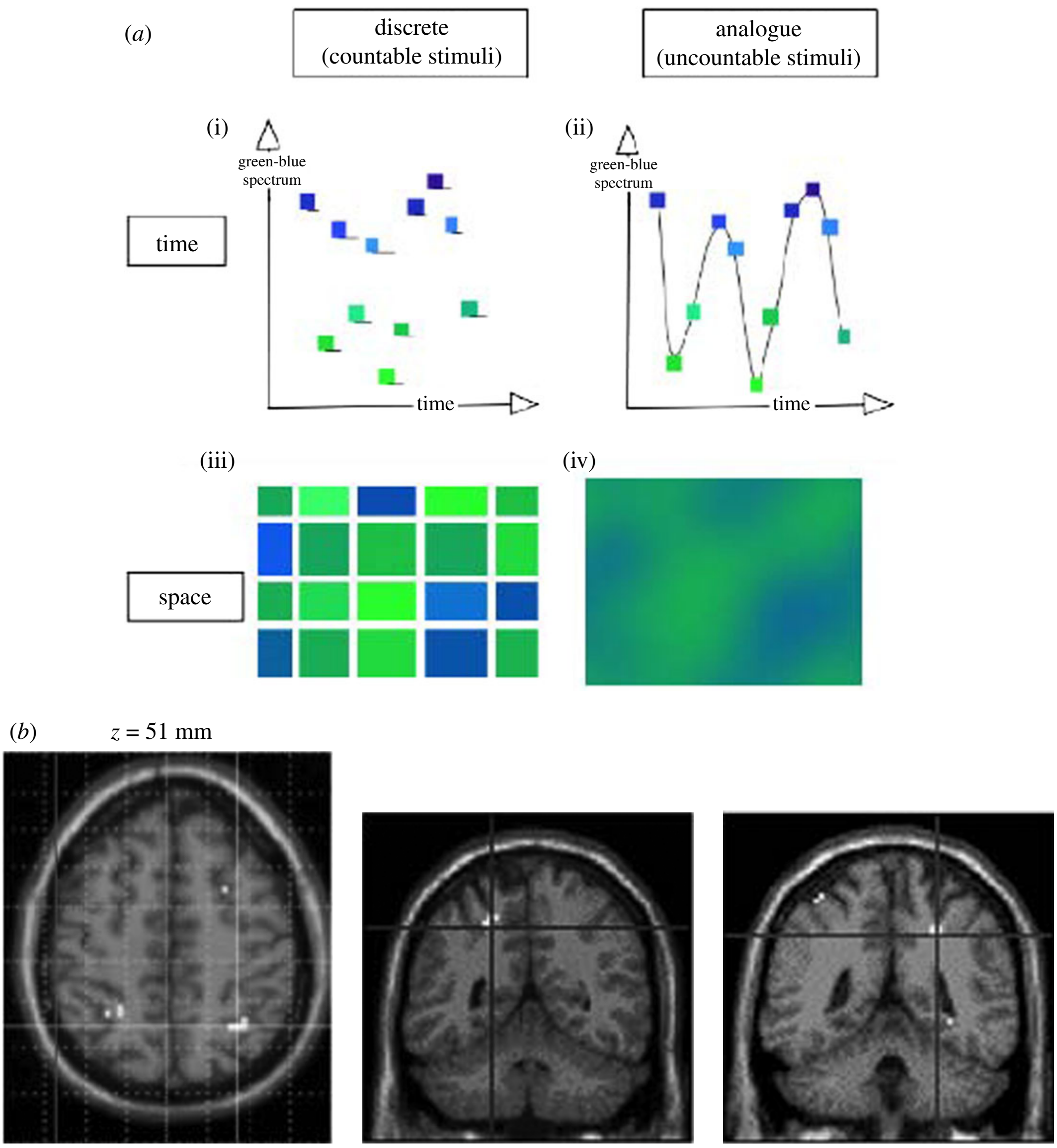

Figure 7. A task to identify brain activations specific to numerosity comparison as opposed to a comparison of any kind of quantity. (*a*) The fMRI task requires a button press to decide whether more blue or more green has been observed. The same hues are used in all conditions. (i) In the discrete temporal condition, a sequence of blue and green squares appears at random intervals between 150 and 400 ms in the same place on the screen. (ii) In the corresponding analogue temporal condition, the same hues are linked by intermediate hue values, so that a single square appears to be smoothly changing hue. In the corresponding discrete spatial condition (iii), the same hues are formed into discrete rectangles separated by a grey background, whereas, in the analogue spatial condition (iv), a smoothing function blurs the boundaries between the different hues. Every trial of discrete stimuli is transformed into a trial of analogue stimuli. Task: is there more green or more blue? (button press response). (*b*) Conjunction analysis (spatial and temporal presentations). Bilateral areas along the length of the IPS were more activated by numerosity processing (discrete, countable stimuli) than extent processing (analogue, uncountable stimuli). Numerosity processing activates the IPS bilaterally. The activity in the IPS increased as the ratio difference between the number of green and blue stimuli decreased in both time and space conditions. Activations specific to the numerosity judgement and modulated by the ratio of blue to green rectangles [64].

By contrast, our study of 11 562 children in Havana found that the core deficit as measured by DE had a male : female ratio of 2.4 : 1 [63]. Consistent with this, a longitudinal study of primary school children showed that boys were much more likely than girls to have a deficit on the 'Number Sets' task, which involves digits and small sets of objects, and probably taps the core capacity in a similar way to DE [23].

On the other hand, no gender differences were found in a large sample of 16-year-olds when tested on numerosity comparisons similar to figure 4*a* [90], which suggests that this task is less influenced by genetics than DE.

It is not clear why some studies find sex differences, and others do not.

(b) Twin studies

The first study to use twins to estimate the heritability of numerical abilities and disabilities was carried out by a team at the Colorado Learning Disabilities Research Center led by John de Fries and Bruce Pennington, pioneers in this kind of work. Taking twin probands as being 1.5 s.d. below the mean on a standardized arithmetic and comparing them with MZ and DZ co-twins, they found a significant groupwise (h^2g) heritability of about 38%, controlling for verbal, performance and full-scale IQ [91].

The most extensive twin study comprised a sample of 1500 pairs of MZ twins and 1375 pairs of DZ 7-year-old twins [83,92]. Their mathematical ability was based on the teacher's assessment in relation to National Curriculum Key

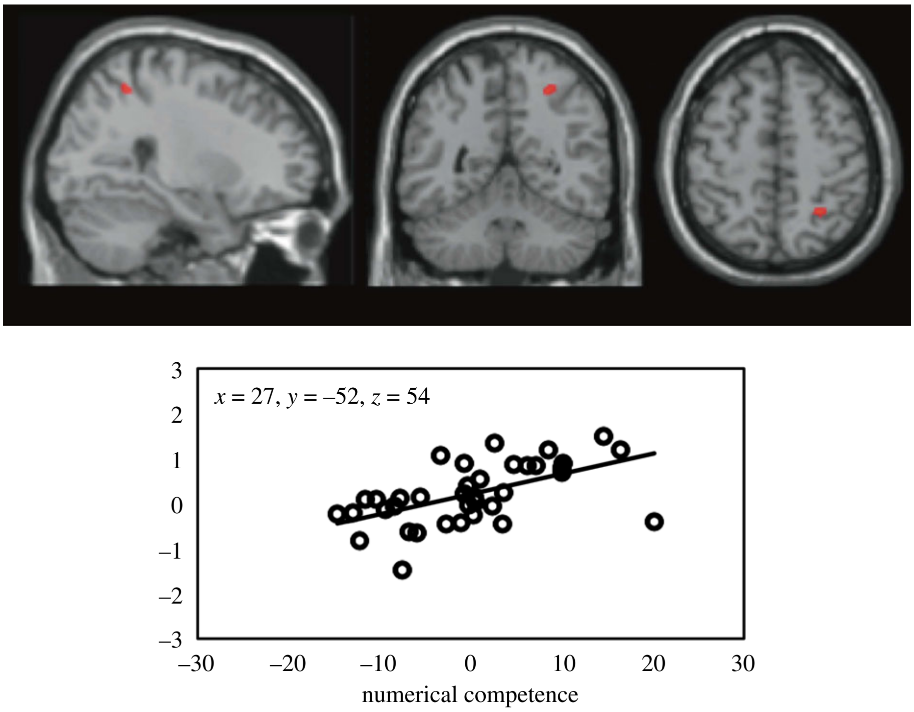

Figure 8. The relationship between numerical competence and a small region in the right intraparietal sulcus [31]. Figure 1 in §2 presents the numerical tasks used in this study. This figure shows the positive correlation by participant between normalized performance of the numerical tasks and activation in the right IPS. (Online version in colour.)

Stage 1. Teachers used a 5-point scale based on the teacher's knowledge of the child's mathematics achievement over the academic year on three aspects of mathematical ability: using and applying mathematics; numbers; and shapes, space and measures. Reading and general cognitive abilities were tested using standardized test batteries. They found a significant proportion of the genetic variance—about 30%—was specific to mathematics.

A second investigation by Kovas and her team used a Web-based battery of tests of maths and reading on 2596 pairs of 10-year-old twins from the Twins Early Development Study [93]. Here, as with Alarcón *et al.* [91], they selected a subsample of children who were particularly bad at maths (the lowest 15%). This required a different method of statistical analysis (DeFries–Fulker Extremes Analysis, h^2g) [93]. They concluded that 'Both reading and mathematics disability are moderately heritable (47% and 43%, respectively) and show only modest shared environmental influence (16% and 20%) . . . [but there was a] genetic correlation of 0.67 between reading disability and mathematics disability, suggesting that they are affected largely by the same genetic factors.' (p. 914).

When the 4000 twin pairs from the Twins Early Development Study reached the age of 12, a further investigation was carried out, again looking at the lowest 15% in reading and maths, along with tests of language and general cognitive ability (IQ). Again, there was a significant genetic influence on maths disability, as well as on reading, language and IQ [94].

Now, these are studies of poor numeracy in general and not of the core deficit specifically. To explore this, the team carried out a more recent study specifically of the heritability of 'number sense'. Kovas and co-workers [90] tested 4518 twins (2259 pairs): 836 monozygotic (MZ), 733 dizygotic same-sex (DZ) and 689 dizygotic opposite-sex (DZ) pairs (sic), with a mean age of 16.6 years, drawn from the Twins Early Development Study. The number sense test they used was the same as the one reproduced in figure 4*a* above. The study found that number sense was 'modestly heritable (32%), with individual differences being largely explained by non-shared environmental influences (68%) and no contribution from shared environmental factors.' ([85], p. 35).

In our study of 160 twins with a mean age of 12 years, we found that the efficiency of DE (median RT/accuracy) was modestly heritable ($h^2 = 0.47$). We also found that grey matter density in the left IPS was modestly heritable ($h^2 = 0.28$), but perhaps what was most interesting for the core-deficit hypothesis was the heritability of the *link* between the efficiency of DE and the efficiency of simple arithmetic, for which our bivariate genetic analysis revealed a respectable heritability of 0.54 [95].

It is fair to say that the genes responsible for the inheritance of mathematical ability and of the efficiency of the core capacity have not yet been established. Candidate gene variations have been found to account for tiny effects in mathematics [96], or have failed to replicate [97]. None so far have been found that are specific to core capacity.

In figure 9, I try to summarize the findings presented above. At the behavioural level, there are selective disabilities in numerical abilities that cannot be explained in terms of environmental or domain-general cognitive factors. At the cognitive level, I have proposed a specific mechanism—an accumulator—a numerosity tool that is part of the starter kit for learning arithmetic. I have called this the 'core capacity' and I have argued that easily measured inefficiencies in this mechanism, a 'core deficit', are one important cause of low numeracy. At the neural level, this core capacity is located in the IPS of the human brain, and in cases of dyscalculia, both the structure and functioning of this mechanism are abnormal and result in a 'core deficit'. Although there is clear evidence for the inheritance of specific

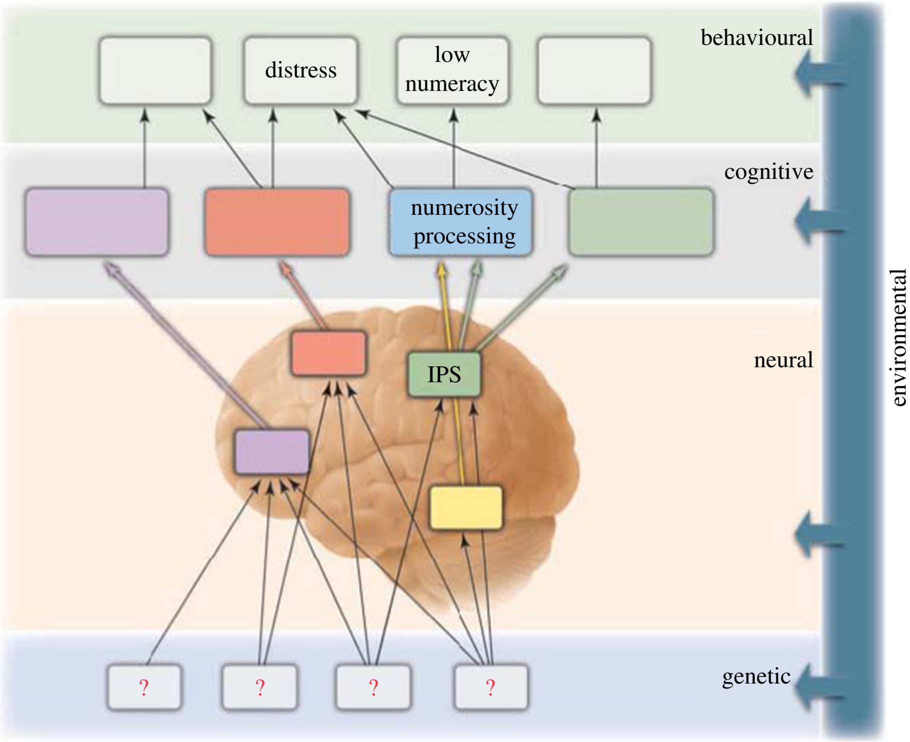

Figure 9. Levels of explanation summarizing the principal elements of the innate numerosity-processing mechanism (after [98] using the 'causal modelling framework' [1]). (Online version in colour.)

numerical abilities from twin and gender studies, so far only one study has explored the genetic link between the core capacity and arithmetical competence. There is still no clear evidence for the genes that could be involved.

6. Educational implications

It is widely attested, and indeed widely understood, that many learners fail to attain the minimum level of competence needed in a numerate society, defined by PISA, for example, as 'Level 2'. As I described in §2, PISA results corroborate the findings from national studies. I will show that low numeracy is important not only for the life chances of individuals but also for society as a whole.

(a) Implications for individual learners

The implications for individual life chances of a large British 1970 cohort are summarized as in the report *Does Numeracy Matter More?*:

> An earlier study, 'Does Numeracy Matter?' . . . showed that people with poor numeracy tended to leave full-time education at the earliest opportunity and usually without qualifications, followed by patchy employment with periods of casual work and unemployment. Most of their jobs were low skilled and poorly paid and offered few chances of training or promotion. . . . Overall, poor numeracy rather than poor literacy was associated with low economic well-being. [28, pp. 4–6]

Participants with low numeracy irrespective of their literacy also had less chance of being in a company pension scheme; were more at risk of depression; and were more likely to have been suspended from school, or arrested and cautioned by the police. So yes, low numeracy does matter for individuals.

Low numeracy in school can be very distressing and can cause problems in the classroom. We asked 9-year-olds about their experiences with the daily numeracy hour. Instead of one-on-one interviews, we arranged the children into five-person focus groups defined by their arithmetic attainment level and led by my colleague, Anna Bevan. We recorded the sessions and made verbatim transcriptions from recordings [99]

— Low attainer: 'When I don't know something, I wish that I was like a clever person and I blame it on myself'.
— Low attainer: 'I would cry and I wish I was at home with my mum and it would be—I won't have to do any maths'.
— High attainer about low attainers: 'They waste their time crying'.
— High attainer about low attainer: 'She's like—she's like all upset and miserable, and she don't like being teased'.
— High attainer about low attainer: 'She goes hide in the corner—nobody knows where she is and she's crying there'.

These learners are a problem also for the teachers. In this case, teachers were interviewed individually by Bevan.

— Teacher KD: 'If they forget really basic things from the beginning, then there's no way you can use those further down the line . . . because they can't even do the basics'.

Teachers are generally aware that children who were struggling often try to hide the fact:

— Teacher JL: '. . .lots of times they're trying to cover it up . . . sometimes they'll cover it up—they'd rather be told off for being naughty than being told off that they're thick'.

As noted in §1, there are many reasons for very low numeracy, but the results reported in the previous sections imply that a proportion of the low numerates suffer a *core deficit* in a mechanism for identifying and representing numerosities, and that this mechanism is a key tool in the *starter kit* for learning arithmetic.

A large and authoritative UK Government Office of Science report, *Mental Capital and Wellbeing* in 2008, one of the few official reports to recognize dyscalculia, summarized the situation thus: 'Developmental dyscalculia is currently the poor relation of dyslexia, with a much lower public profile. But the consequences of dyscalculia are at least as severe as those for dyslexia' [100, p. 1060]. It notes that dyscalculia can reduce lifetime earnings by £114 000 and reduce the probability of achieving five or more GCSEs (A*–C) by 7–20 percentage points, both of which are significantly worse than the impact of dyslexia [100,101].

(b) Implications for society

The prevalence of this condition—which is usually termed 'developmental dyscalculia'—has been estimated as somewhere between 3.5% [63] and 7% [102]. Putting these estimates into context, this means that in the UK there are between 2.1 and 4.2 million sufferers, and in the USA, between 10 and 20 million sufferers. (Bear in mind that these are studies of school children and are not strictly comparable with the cohort studies of adults, and it is possible that there are cumulative effects of low attainment at school. Low maths attainment at school typically leads to earlier school leaving [28] and probably to an even greater competence gap at adulthood with more competent peers.)

I have argued that a significant proportion of all learners who have fallen behind their peers will have a core deficit. The estimate in our Havana study was that about *one-third* of those with 'arithmetic dysfluency'—essentially, those 9% of children who are slow or inaccurate on timed arithmetic—have a core deficit and are dyscalculic [63]. Geary *et al*. estimate that learners with 'mathematical learning disability'—a broader category than our AD—who have a core deficit as measured by their 'number sets' task, represent about 10% of all the 'low achievers' [23]. As mentioned in §1, dyscalculics represent a small but significant proportion of all learners who are regarded as having low numeracy, and not just those characterized as dysfluent in the Havana study. Using the PISA criterion reference, this amounts to 23% in OECD countries, so those with dyscalculia constitute one-third to one-fifth of these low numerates.

One implication for society is the cost of dyscalculia. The accountancy firm, KPMG, estimated the cost to the UK of the lowest 6%, in terms of lost direct and indirect taxes, unemployment benefits, justice costs because they are more likely to be in trouble with the law, medical costs because they are more likely to have physical or mental problems, and additional educational costs, was £2.4 billion per year [103].

The additional educational cost was calculated at £235.2 million, one-tenth of the total cost. These costs are specified in table 1.

Table 1. Total lifetime costs for the annual cohort of 35 843 children with numeracy difficulties (in £millions), from [103].

educational cost category	total lifetime costs (in £millions)
special needs support – numeracy (primary)	51.5
special needs support – numeracy and behaviour (secondary)	90.5
cost of maintaining a statement of special educational needs	83.4
educational psychologist time	4.1
permanent exclusions	0.9
truancy	2.8
adult numeracy classes	2.0

(c) Policy

One cannot help think that the total cost to society would be lower if more were spent on educational help for the lowest-attaining, especially the dyscalculics. Research into the best way of identifying and helping dyscalculics will require funding levels at least comparable with the funding for dyslexia research. However, NIH funding in 2009 in the USA for research into dyscalculia was 6% of that for dyslexia, and a similar proportion of publications [104]. Comparable figures for the UK and other countries do not yet exist.

The fact that neuroscience has identified a target deficit does not entail how this deficit should best be ameliorated, any more than the identification of a disease target entails the precise nature of a drug or how it should be administered.

Now, what should these appropriate educational interventions be? There are well-established practices for helping dyslexic learners to read and spell, but there is nothing comparable for dyscalculics. There are two critical features of literacy interventions for dyslexia: multisensory methods and personalized learning plans. Special needs teachers who specialize in helping dyscalculic learners in the UK describe their own multisensory methods and their focus on designing interventions for each individual learner [105–107]. However, these methods have not been subject to systematic evaluation.

In fact, the present UK government and its immediate Conservative predecessors do not officially recognize dyscalculia. The previous references have been removed, and now the only reference on the Department for Education website is to the Driver and Vehicle Licensing Agency, where it is stated that dyscalculia does not prevent someone from getting a driving licence. There is also a link to the Department for Work and Pensions about discrimination in employment.

The UK DFEE (Department of Education and Employment) in *Numeracy Matters* stressed that in initial teacher training, teachers 'need to have sufficient knowledge and skills to teach numeracy well to primary pupils' (para 73, [6]). However, although the report drew attention to children's special educational needs, and the fact that children with numeracy problems should have an Individual Education Plan, it said nothing about dyscalculia or any equivalent formulation of specific numeracy disabilities. In fact, there are still very few courses to teach teachers about dyscalculia. We set up the first one in the London Borough of Harrow in 2003 funded by a local charity, the John Lyon Trust, which ran for a year. In a survey by the British Dyslexia Association's Dyscalulia Committee in 2015, only two universities were identified that had courses targeted at dyscalculia (www.bdadyslexia.org.uk/dyslexic/dyscalculia). It is of course important to have expert assessment and guidance for learners with dyscalculia and other types of severe difficulties with numeracy. However, it is difficult to find any postgraduate courses in educational psychology or in mathematics teaching that provide training in this area.

National Numeracy is a UK charity that wants 'Everyone in the UK to have the numeracy that allows them to make the most of their lives'. The only reference on their website to dyscalculia is the following: 'A senior lecturer for the Dyslexia Programme at Edge Hill University has shared her recommendations for the top ten dyscalculia books.' (https://www.nationalnumeracy.org.uk). National Numeracy makes no use of the extensive behavioural, cognitive, neuropsychological and genetic research on dyscalculia published in over three hundred peer-reviewed papers since 2007. In fact, National Numeracy claims that 'Mathematical understanding is not determined at birth', without any recognition of individual differences in the starter kit, especially with regard to foundational numerosity concepts.

(d) Individualizing learning

The UK government's report *Mental Capital and Wellbeing* (2008) [100] specifically calls for individual education plans for dyscalculia. Both this report and *Numeracy Matters* [6] propose that ICT can provide individualized learning environments by keeping accurate track of the learner's current competence and rehearsing relevant material. Digital interventions, the report notes, need more development. Beyond this, the report has nothing specific to say about the interventions [100,101]. Meta-analyses of the effectiveness of different types of intervention for children with 'special educational needs' [108], 'learning disabilities' [109] or of 'fact-based dysfluency' [110] fail to distinguish the different types of learning disorder, so they cannot provide a basis for personalized intervention strategies that is matching the appropriate intervention for the learner's particular problem. In particular, they do not distinguish between dyscalculia and other causes of low numeracy.

Randomized controlled trials with one digital game, 'Number Race', have also been promising with early school children [111] and preschoolers [112], though these studies are not specifically targeted at learners with low numeracy or with dyscalculics. Other games are currently available, but not yet fully evaluated (e.g. http://www.thenumbercatcher.com/nc/home.php).

Our own attempts to use adaptive digital technologies suggest that they provide the opportunity for much more practice in what Vygotsky has termed the 'zone of proximal development'—that is, just beyond the level of current competence—and where the program acts as a tutor, giving tasks and feedback adapted to their current needs [113,114]. We also try to go beyond merely rehearsing concepts that are already known, or what *Numeracy Matters* [6] calls 'practice and reinforcement' (para 54), because the problem for these learners is to understand the concept in the first place. Rather, we adopt constructionist pedagogical theory [115] using a simple microworld [116] to support children learning the foundational concepts and principles of the structural properties of numbers, such as the commutativity of addition and the relationship between addition and subtraction. It is not sufficient to learn a list of arithmetical facts.

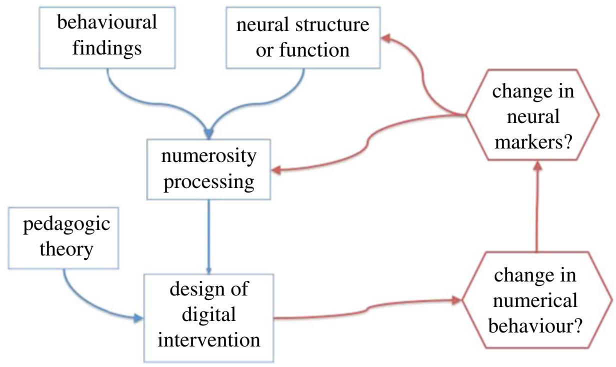

Figure 10. Sketch of an iterative research programme. The intervention is designed to modify the core cognitive deficit in numerosity processing identified by individual differences in behaviour, and in neural markers, where tested. If the intervention is effective, then there will be changes in neural structure or functioning, as well as in numerical behaviour, where both are important measures of the effectiveness of the intervention in ameliorating the core deficit. This programme is iterative, with feedback from changes in brain and behaviour modifying the design of the intervention (not shown with arrows). (Online version in colour.)

These activities are targeted specifically at the core deficit identified in the neuroscience, and the constructionist pedagogy gives dyscalculics more of the relevant practice they need to strengthen the mental representation of numerosities, the relationship among numerosities, and their relationship to the familiar words and numerals. This is comparable to the way that dyslexics benefit from training targeted at their core deficit in phonological processing [117–120] and its relationship to letters and words [121]. The research programme we advocate is summarized in figure 10.

Figure 10 depicts a research programme in educational neuroscience in which digital interventions targeted at the core deficit are tested against both behavioural and neural changes [113,122,123]. Individual differences in response to intervention would be an important element in the iterative design process.

This research programme requires that educational authorities, policymakers, parents and sufferers recognize that poor numeracy can have a very specific cause and will need very specific help.

Data accessibility. This article has no additional data.
Competing interests. I declare I have no competing interests.
Funding. I received no funding for this study.

Endnote

[1]There was also a test of timed number comparison but this was less discriminating than DE, so it is not included here.

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