Numerical thought with and without words: Evidence from indigenous Australian children

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Are thoughts impossible without the words to express them? It has been claimed that this is the case for thoughts about numbers: Children cannot have the concept of exact numbers until they know the words for them, and adults in cultures whose languages lack a counting vocabulary similarly cannot possess these concepts. Here, using classical methods of developmental psychology, we show that children who are monolingual speakers of two Australian languages with very restricted number vocabularies possess the same numerical concepts as a comparable group of English-speaking indigenous Australian children.

cognitive development | linguistic determinism | mathematical cognition | number concepts

A strong form of the hypothesis that language determines thought (the Sapir-Whorf hypothesis) (1) has been revived for the domain of numbers (2, 3) on the basis of studies of Amazonian cultures with languages that lack counting words (4, 5), even though the hypothesis has been largely abandoned elsewhere (6,7). It is argued that a vocabulary of counting words is necessary for a person to possess the concepts of exactly four, exactly five, and so forth (2, 3). Without the vocabulary, only primitive, approximate numerical values are possible (2, 8). It is proposed that counting words modify two innate core systems of knowledge with numerical content (2,8): parallel individuation of objects, which enables the representation of exact numerosities up to three, and analogue magnitudes that represent approximate numerosities of more than three. According to one version, children learn to associate the words "one," "two," and "three" with the state of the parallel individuation system, and generalize from this that other number words also denote exact numerosities (2). On another account, children make use of the fact that they have already associated larger number terms with approximate numerosities (9), and refine their sense of, for example, approximately fiveness into exactly fiveness (5,10). It follows from these accounts that a child raised without linguistic means for representing increasing exact numerosities will not be able to develop concepts of the natural numbers, each denoting an exact numerosity with a unique successor.

Evidence for the strong form of Whorf's hypothesis comes from two studies of the numerical abilities of speakers whose languages have restricted number-word vocabularies. Adult speakers of Pirahã, an Amazonian language that contains words for just "one," "two," "few," and "many," have difficulty putting small sets of objects in one-to-one correspondence, and fail in a task working out the consequence of adding to, or subtracting one item from, a small set of objects (4). Another Amazonian group, the Mundurukú, whose language contains words for exact numbers to about three and approximate numbers to about five, perform comparably with French adult controls on tasks involving approximate numerosities, but are much worse than controls on simple exact subtraction (5). Both Amazonian groups are hunter-gatherers whose lifestyles differ from our own in many ways, but the factor held responsible for the difference on number tasks is their limited vocabulary of number words (11).

In the study we report here, we contrasted three languages: Warlpiri, Anindilyakwa, and monolingual English. Warlpiri is a classifier language spoken in the Central Desert north and west of Alice Springs, Northern Territory (NT). It has three generic types of number words: singular, dual plural, and greater than dual plural. Anindilyakwa, another classifier language, is spoken on Groote Eylandt, NT, in the Gulf of Carpentaria. It has four possible number categories: singular, dual, trial (which may in practice include four), and plural (more than three) (12). There are also loan words used as number names for 1, 2, 3, 4, 5, 10, 15, and 20, but these appear to be used only in certain contexts and children do not know them (13). Neither language has ordinals equivalent to "first," "second," "third"; both have quantifiers similar to "few" and "many" (12). [For further details about both languages, see supporting information (SI) *Text*]. We also tested monolingual English-speakers in Melbourne, Australia at a school for indigenous children. See Fig. 1 for NT locations.

Results

We tested 45 children aged 4 to 7 years old: 20 Warlpiri-speaking children, 12 Anindilyakwa-speaking children, and 13 English-speaking children from Melbourne. Approximately half the NT children were 4 to 5 years old and half were 6 to 7 years old. We used four enumeration tasks to evaluate numerosity understanding: memory for number of counters, cross-modal matching of discrete sounds and counters, nonverbal exact addition, and sharing play-dough disks that could be partitioned by the child (see *Methods* and *SI Text*)

Memory for Number of Counters. No language effects were found ($F < 1$) (see Fig. 2*A*). Children were more accurate recalling small, compared to large numerosities [$F(1, 24) = 16.05$, $P < 0.001$], and older NT children recalled more than their younger peers [$F(1, 28) = 16.30$, $P < 0.001$]. No other effects were found.

Cross-Modal Matching. No language effects were observed ($F < 1$ Fig. 2*B*). Young children in all locations were more accurate at cross-modal matching small compared to large numerosities [$F(1, 18) = 14.82$, $P < 0.001$]. Older NT children were more accurate than their younger peers [$F(1, 25) = 5.41$, $P < 0.03$]. No interactions were observed.

Nonverbal Addition. Although Melbourne children solved fewer problems correctly than their NT peers, the difference was not significant ($P > 0.1$) (Fig. 2*C*). Children solved more simple than

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Fig. 1. Location map of Willowra and Angurugu.

difficult problems [F (1, 17) = 15.34, $P < 0.001$ for young children in all locations; F (1, 19) = 13.96, $P < 0.001$ for the two NT groups]. Older NT children solved more problems than their younger peers [F (1, 19) = 13.38, $P < 0.001$]. A significant interaction between set size and age [F (1, 19) = 4.67, $P < 0.05$] was attributable to the younger children's relative inability to solve the more difficult problems ($P < 0.05$).

Sharing. Almost all children were able to share six or nine items among the three bears, typically using one-to-one dealing (Fig. 2*D*). Children were less successful sharing seven and ten items: only some older NT children successfully partitioned and distributed the spare play dough disk appropriately. [7 items χ^2 (2, n = 33) = 8.07; 10 items χ^2 (2, n = 33) = 6.12, p's < .05].

To understand how exact number was varied as a function of individual differences, hierarchical cluster analyses (using Ward's method) were performed on each measure to identify task-related low, medium, and high performances. Tables 1 and 2 show that children with the highest level of competence were from all sites.

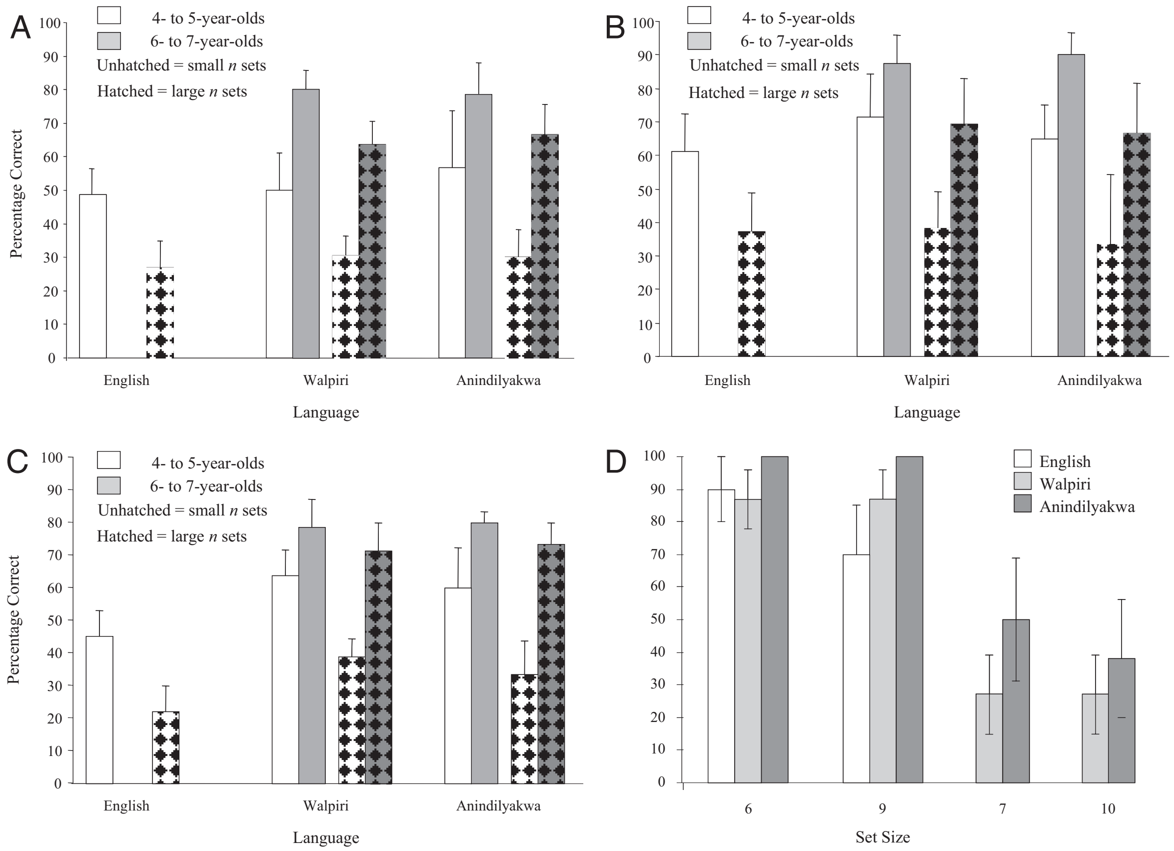

Fig. 2. Percentage of correct responses as a function of age, set size and language for the memory for counters (*A*), cross-modal matching (*B*), nonverbal addition (*C*), and sharing (*D*) tasks.

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There were significant relationships between performance and age on the memory for counters, nonverbal addition, and sharing measures. Older children were over-represented in the high performance groups [memory for counters: χ^2 (2, n = 45) = 12.82, $P < 0.002$; nonverbal addition: χ^2 (2, n = 32) = 12.77, $P < 0.002$; sharing: χ^2 (2, n = 33) = 6.88, $P < 0.03$].

We also analyzed the relationship between responses and targets to determine whether there was a discontinuity between small ($\leq$4) and large numbers (>4) (Fig. 3 *A–C*). There was a linear trend for each language group for all tasks, with r^2 values between 0.80 and 0.99 and no observed discontinuities between small and large numbers (see *SI Text* for linear trends.) MANOVAs, adjusted for age, confirmed that there was no difference between groups. The scalar variability of responses (coefficients of variation) was not significantly different from zero for the tasks in all language groups (see *SI Text*). This is consistent with the use of nonverbal enumeration, but not verbal counting, for all numerosities in these tasks (14).

In this study, no language effects were observed. Neither the Warlpiri-speaking nor Anindilyakwa-speaking children performed worse than the English-speaking children on any task. Failure to find performance differences was not because of the

Table 1. Number of children assigned to low, medium, and high competence cluster groups as a function of task and language

Task and language	Low	Medium	High
Memory for counters			
English	8	2	3
Warlpiri	7	6	7
Anindiyakwa	4	4	4
Cross-modal matching			
English	3	5	1
Warlpiri	2	8	8
Anindiyakwa	1	6	3
Nonverbal addition			
English	5	3	1
Warlpiri	5	1	7
Anindiyakwa	2	2	6
Sharing continuous quantity			
English	3	7	
Warlpiri	1	10	4
Anindiyakwa		5	3

Table 2. Number of children assigned to low, medium, and high competence cluster groups as a function of task and age

Task and age	Low	Medium	High
Memory for counters			
4 and 5 year olds	17	7	3
6 and 7 year olds	2	5	11
Cross-modal matching			
4 and 5 year olds	4	13	5
6 and 7 year olds	2	6	7
Nonverbal addition			
4 and 5 year olds	11	6	5
6 and 7 year olds	1		9
Sharing continuous quantity			
4 and 5 year olds	3	15	1
6 and 7 year olds	1	7	6

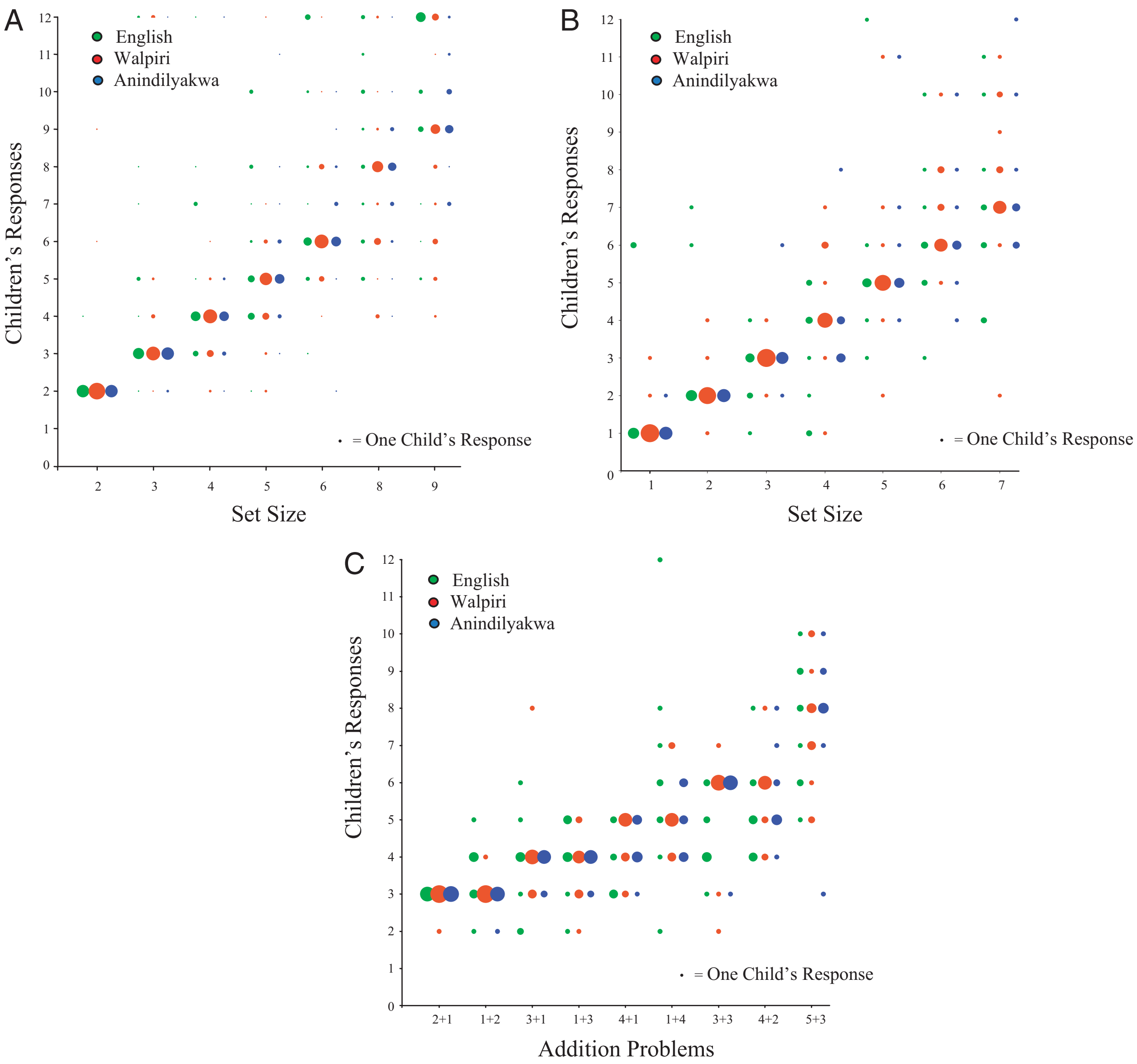

Fig. 3. Distribution of children's responses as a function of set size for the memory for counters (*A*), cross-modal matching (*B*), and nonverbal addition (*C*) tasks.

insensitivity of the tests, as one predictor variable significantly related to performance: namely, children's age. If a number vocabulary were necessary for the development of exact number concepts, then no NT children should have achieved high levels of numerical competence, yet high levels were reached by both NT groups in all tasks (see Fig. 4 *A* to *C* for response frequency distributions). Discontinuities in accuracy between small and large numbers is held to be the signature of the two core systems operating without the aid of language (2, 3, 8). Here we showed that tasks became harder as the number increased, with no discontinuities in the linear trend from 1 to 9 objects for all groups (see *SI Text*).

Methodological differences between this study and the Amazonian studies may account for the conflicting results. The adults in the Pirahã study (4) may not have understood the tasks (15), and in the Mundurukù study (5) subtraction was the only exact number task, and this operation is difficult for children 4 to 5 years of age (16) (see *SI Text*).

We conclude that the development of enumeration concepts does not depend on possession of a number-word vocabulary. Alternative accounts propose that we are born with a capacity to represent exact numerosities (17–19), and that using words to name exact numerosities is useful but not necessary (11, 20). When children learn to count, they are learning to map from their pre-existing concepts of exact numerosities onto the counting word sequence (11, 20). Conceptual development drives the acquisition of counting words rather than the other way around.

Methods

In Willowra and Angurugu, bilingual indigenous assistants were trained by an experimenter (D.L. or F.R.) to administer the tasks, and all instructions were given by a native speaker of Warlpiri or Anindilyakwa. Procedures used to familiarize NT children and indigenous assistants with tasks and materials are described in the *SI Text*. Piloting showed children could easily grasp the purpose of the tasks. The experimenter recorded relevant aspects of task performance as they occurred.

Memory for Counters. Identical 24-cm × 35-cm mats and bowls containing 25 counters were placed in front of a child and the experimenter. The experimenter took counters from her bowl and placed them on her mat, one at a time, in preassigned locations. Four seconds after the last item was placed on

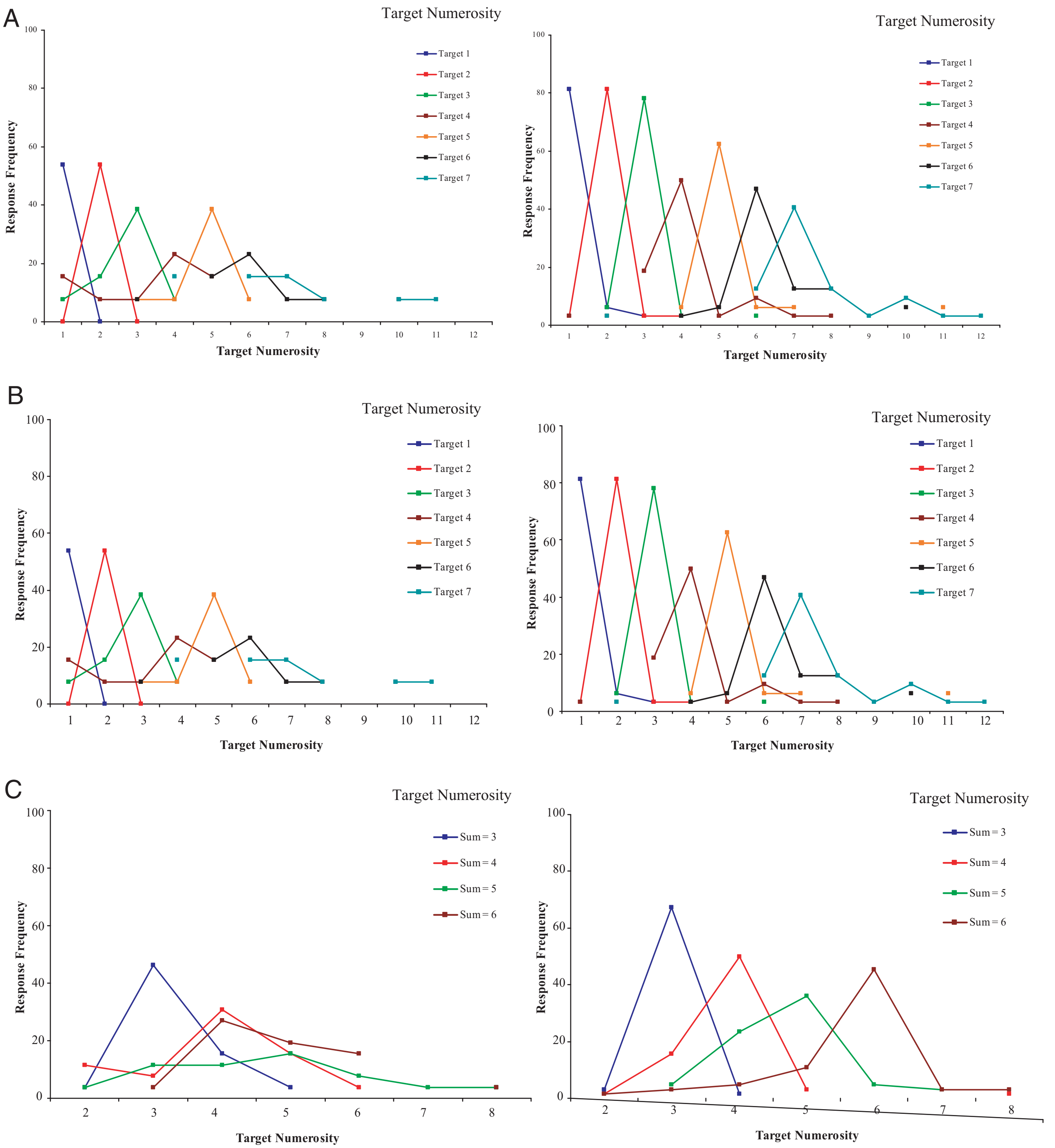

Fig. 4. Target numerosity frequency graphs for English-speaking (*Left*) and combined indigenous language groups (*Right*) for the memory for counters (*A*), cross-modal matching (*B*), and nonverbal addition (*C*) tasks.

the mat, all items were covered with a cloth and children were asked by the indigenous assistant to ''make your mat like hers.'' Following three practice trials in which the experimenter and an indigenous assistant modeled recall using one and two counters, children completed 14 memory trials comprising two, three, four, five, six, eight, or nine randomly placed counters. In modeling recall, counters were placed on the mat without reference to their initial location. Number and locations of children's counter recall were recorded.

Cross-Modal Matching. The experimenter demonstrated the task by tapping two wooden blocks once and placing a single counter on the mat, while the indigenous assistant said, ''Like this? Yes!'' The experimenter then tapped the blocks three times and placed three counters on her mat—the indigenous assistant said: ''Like this? Yes!'' The experimenter tapped the blocks three times again, but placed only two counters on the mat. The indigenous assistant said: ''Like this? No!'' The experimenter placed a third counter on the mat and the indigenous assistant said: ''Like this? Yes!'' Seven trials, comprising numerosities one to seven, were presented in a random order.

Nonverbal Addition (21). Using materials from the memory task, the experimenter placed one counter on her mat and, after 4 seconds, covered her mat.

Next, the experimenter placed another counter beside her mat and, while the child watched, slid the additional counter under the cover and onto her mat. Children were asked by the indigenous assistant to ''make your mat like hers.'' Nine trials comprising 2 + 1, 3 + 1, 4 + 1, 1 + 2, 1 + 3, 1 + 4, 3 + 3, 4 + 2, and 5 + 3 were used. Children's answers were recorded.

Sharing. This task assessed the ability to share quantities of play-dough among three toy bears. Although the play-dough disks comprised equal-sized discrete units (3-cm disks), each disk-unit could be regarded as a continuous quantity for sharing purposes. Following two practice trials in which children shared four disks between two bears (''give these to the bears''), they completed four randomly-ordered trials comprising 6, 9, 7, and 10 disks, which they shared among the three bears. The experimenter recorded the number of disks given to each bear, the sharing strategies, and the treatment of any remainder disk; that is, whether it was given to one bear or an attempt was made to divide it among the three bears.

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