Chapter 10.

From fear of fractions to the joy of maths

Here's a question. "What is a half of three-quarters?" I ask this question not because it's particularly hard but because this was a question put to the former Chief Inspector of Schools in England (1994 - 2000), Mr Chris Woodhead on a radio show. Mr Woodhead had been complaining about falling standards, especially in maths. This was a concern shared by many parents and teachers. One teacher phoned the radio show, complaining that the 16-year-olds she taught were hopeless. They couldn't even solve simple problems like "what is a half of three-quarters". The host of the show did not ask the Chief Inspector to comment on this piece of news. Instead, she asked him for the answer to this question. Remembering the embarrassment an education minister had recently suffered in a similar situation (see Chapter 8), he refused to answer. Later a spokesman for the Inspectorate told the world that the Chief Inspector knew the answer but didn't want to say. In any case, his speciality was English, not maths.

No doubt, the Chief Inspector did know the answer, but like many people, lacked confidence in the answer. Why is this? Confidence comes from knowing what you're doing – and also knowing that you know what you're doing. The way many of us are taught doesn't help us gain confidence. For the fraction problem that may have defeated the Chief Inspector of Schools, most of us learned (or failed to learn) a procedure that would give the right answer. Multiply the denominators (2 x

4) to make the new denominator, and multiply the numerators (1 x 3) to make the new numerator. Lo and behold you have the answer 3/8. But can you be sure this is the right answer? How can you have confidence?

One of the great beauties of mathematics – and what makes it unlike most subjects we learn at school – is that there are many ways to find the answer. So you will have confidence if you arrive at the same answer using a different procedure. But few of us have learned other procedures.

Another path to confidence is to make an estimate and check that the answer is consistent with the estimate. To do this, you need to understand the problem. Half of something is less that the whole of it, so half of 3/4 is less than 3/4. What is more, because 1/2 of 3 is 11/2 the answer has to be one and a half quarters, that is, more than one quarter and less than two quarters. You could also see that 1/4 = 2/8, and that half of 2/8 is 1/8.

In England, and probably in Japan, teachers and other educational authorities have decided which is the best method for each type of problem, and this is what you learn. As a consequence, most of us lack confidence in our ability to deal with fractions, so we avoid and fear them.

It doesn't have to be like this. And it is never too late to understand fractions or to have confidence that you do. What could be a more severe test of confidence, and competence, than doing your calculations on TV in front of millions of viewers willing you to succeed? This is exactly the situation in which Rüdiger Gamm found himself. He appeared on a German TV programme in which the audience bets whether an expert can accomplish a difficult challenge.

Here's a question he answered very swiftly. What is 87 to the 12^{th} power (87^{12})? [1]

You couldn't do this in your head, could you? You may think this is because you have no special talent for mathematics. But neither has Gamm. At school, he hated maths, and was no good at it. So what happened? At the age of 20, he found a piece of maths he could understand, and with practice, could do. Like the champion calculator, Wim Klein, (Chapter 7), he found he could use this skill to impress. More than that, he saw the possibility of a new path to fame and, as it turned out, to a modest fortune. Of course, this level of achievement comes at a cost. Gamm, who is now known as "the human calculator" and makes a living doing calculations, works four hours a day at his new profession. He has learned tables of squares and cubes, square roots and cube roots, like we once learned our multiplication tables. Just as we can solve 8 times 5 almost instantly, so Gamm can give 27^3 and $\sqrt{169}$ without calculating. But he also knows lots of tricks and short-cuts, so when he is presented with a problem he has many ways to solve it. He can not only check his answer by using two different methods; he can also select the method he has found to be the most efficient. [2]

Most of us can work out the answer to 876 x 458 on paper. We understand what the problem means, we know which steps are needed to do it, and we can carry out each of the steps successfully. For example, we know that 8 x 6 is 48. But most of us could not solve the problem in our heads, because we cannot keep in mind the whole sequence of steps and intermediate results. Gamm has trained himself to do just this. A scientific

1 The answer, in case you have not calculated it, is 188,031,682,201,497,672,618,081

2 see notes

study of Gamm's calculating ability revealed that he uses a part of the brain that the rest of us do not use for calculating. We are limited by our "working memory" capacity, approximately a total of six or seven steps. This, in my case, is about enough to multiply two two-digit numbers such as 45x76, and then I often make mistakes. How then does Gamm manage to exceed this capacity, which is usually thought to be fixed? [3]

Your computer has a random access memory with a relatively small capacity, and a hard drive with much greater capacity. However, your computer can increase the effective capacity of RAM by temporarily offloading data not immediately needed into dedicated "swap space" on the hard drive, retrieving it again as required, creating a "virtual memory" with more capacity than the RAM alone. This is similar to what all kinds of experts seem to be able to do, but only in their domain of expertise. An expert musician can remember the sequence of notes in a new melody much better than a novice. An expert chess player can remember the position from a game at a single glance. An expert waiter can remember the orders from a table of twenty people without writing them down. All these exceed the normal capacity of our "working memory". How is this achieved? It's simple. You become an expert. It so happens, we are all experts in one skill we practice every single day – language. [4]

We are all experts at language. If I give you a sequence of four words from a language unknown to you, and ask you to recall them in the presented order, you will have great difficulty. But you would have no difficulty remembering a list of familiar words. In fact, your capacity for a word list is about 6 or 7, depending on the words. However, if I make the list very much like what you practice every day, make the list into a sentence,

3 Pesenti et al (2001)

4 Butterworth (2001)

then you will have little trouble remembering far more words. Even a thirty-word sentence can be no problem at all. [5] This is because of the hours of practice we have put in to understanding, and, where necessary, remembering what people say to us. We develop very efficient ways of coding the information so that we store it on our internal hard disk – our long-term memory – to be retrieved as needed. No one would be surprised if we were able to demonstrate this skill, and no TV company would pay out a big prize for us to recall a thirty-word sentence. What makes Gamm so unusual is that he chosen to become an expert in what very few us have bothered with.

So could any of us become like Gamm? Most of us wouldn't want spend all that time learning dry arithmetical facts or arcane calculation procedures (give example). But there is no reason to think that we couldn't if we tried. After all, Gamm succeeded, and he seems to have been worse at maths at school than most of us.

Some of us would be put off the whole idea of working away at mathematics, because anything with numbers makes them anxious. As I suggested in Chapter 8, being forced at school to do tasks that one doesn't understand is anxiety-inducing. This is especially true of maths because we are not only forced to do these tasks, but to do them quickly. What is more, learning maths is a process of building one concept on top of another, and, like any kind of building, if the foundations are not secure then the whole edifice will tumble down when too much pressure is applied. Maths tests, even informal ones that occur as part of a lesson, can be seen as a process of applying a stress to the building to see if it is really secure.

5 Wingfield and Butterworth (1984)

There are many paths to enlightenment

As I suggested, one of the things that distinguishes people who are good at maths, have effective "mathematical brains" is an ability to see a problem in different ways. This is because they understand it. This, in turn, allows the use a range of different procedures to solve it, and to select the one that will be most efficient in this particular task. People who are bad at maths stick to one procedure, the one they feel sure about, and use it. [6] Sometimes they will use an inappropriate procedure because that is the only one they know. Take a really simple example: 99 x 14. The weak mathematician would solve this by applying the normal algorithm for long multiplication:

$$\begin{array}{rl} 99 & \times \\ \underline{14} & \\ 396 & + \\ \underline{990} & \\ 1{,}386 & \end{array}$$

This gives the correct answer, of course. The better mathematician would see that 99 x 14 is equivalent to (100-1) x 14, and by a distributive law, to (100 x 14) - (1 x 14), which is easy to work out. But to do this, one needs confidence in one's understanding that the distributive law will give the same, correct answer. It is also rather fun to find a neat re-formulation of the problem, and people who can do this find maths far more enjoyable than those who don't.

The key to Gamm's current exceptional skills lies not in the fact that he spends hours of every day pursuing what has

6 Ostad (1999)

become his profession, but that he suddenly understood some mathematics. What makes us anxious about mathematics or anything else is being confronted with something we don't how to deal with. We don't really know how maths anxiety starts, but it is certainly maintained by feeling that you don't understand what's going on. The cure is simple; make sure practice always follows understanding.

The source of anxiety for many people, I suspect, lies simply in the way that they first experienced mathematics – in how they were taught. If it was all drill and practice, with public displays of failure in the classroom, then it is scarcely surprising that maths and relaxed enjoyment do not go together. Drill also means that you get stuck in one way of doing a problem. You will not have a flexible approach to mathematical problems. What is more, training in computation can let you be captured by the numbers, rather than thinking about the logic of the problem. Here are two examples.

Some animals were loaded on to a boat: four sheep, twenty-seven goats and five horses. How old was the captain of the boat? Silly question? I hope you thought so; but many children (and no doubt some adults) would start to add the numbers together to try to find the answer.

Here's a more sensible question. Imagine a straight wooden pole that is stuck in the mud at the bottom of a pond. There is some water above the mud and part of the pole sticks up into the air. One half of the pole is in the mud; 2/3 of the rest is in the water; and one foot is sticking out into the air. How long is the pole?

This is a problem that was given to 11 year olds in the US. Here are some of their answers.

> First child: You multiply 1/2 by 2/3 and then you add a foot to that.

> Second child: Add the 2/3 and 1/2 first and then add the one foot.
>
> Third child: Add all of them and see how long the pole is.
>
> Fourth child: One foot equals 1/3. Two thirds divided into 6 equals 3 times 2 equals 6

These answers come from normal children educated in a traditional way. They formed part of the control group in perhaps the most radical and startling experiment in education. In 1929, in Manchester, New Hampshire, the superintendent of schools, L. P. Benezet wrote:

> It seems to me that we waste much time in elementary schools, wrestling with stuff that ought to be omitted or postponed until the children are in need of studying it. … I would omit arithmetic from the first six grades [ages 5 to 11 years] … where does an eleven-year-old child ever have to use arithmetic? … What possible needs has a ten-year-old for long division? The subject of arithmetic could be postponed until the seventh year of school, and any normal child could master it in two years study. [7]

Superintendent Benezet was distressed by the children's inability to understand and explain very simple mathematical ideas. He asked them, for example, to explain in their own words that if you have two fractions with the same numerator, the one with the smaller denominator is the larger. Here are some of their answers, recorded by a stenographer:

7 Benezet (1935)

> "The smaller number in fractions is always the largest."
>
> "If you had one thing and cut it into pieces, the smaller piece will be the bigger. I mean the one you could cut the least pieces in would be the bigger pieces."
>
> [Note: these sentences do not make sense.]

Benezet comments that you will think "this must have been a group of half-wits, but I can assure you that it is typical of the attempts of fourteen-year-old children from any part of the country to put their ideas into English. The trouble was not with the children or with the teacher, it was with the curriculum."

Over the next few years, Superintendent Benezet persuaded half the schools in his district to adopt a new curriculum with no formal arithmetic until the children were eleven. There was emphasis, however, on expressing oneself in speech and writing, and on reasoning.

The experiment was an astounding success. He gave the stick problem to a fifth grade class who had had no formal drill in arithmetic, but much mental work on reasoning. Their answers were altogether more sensible.

> "First child: You would have to find out how many feet there are in the mud.
>
> And what else? [Asked Benezet]
>
> Another child: How many feet in the water and add them together? …

> The next child went on to say, One-half of the pole is in the mud and one-half must be above the mud. If 2/3 is in water, then 2/3 and one foot equals 3 feet, plus the 3 feet in the mud equals 6 feet.
>
> The problem seemed very simple to these children who had been taught to use their heads instead of their pencils."

A team from Boston University gave tests to 200 sixth graders, half taught with the new curriculum with no arithmetic until beginning the sixth grade. At the start of the year, those taught traditionally did better on the tests, but by June, the end of the school year, one of the experimental groups led the city. Benezet wrote, "These children, by avoiding the early drill on combinations, tables, and that sort of thing, had been able, in one year, to attain the level of accomplishment which traditionally taught children had reached after three and one-half years of arithmetical drill."

Teaching, in other words, can rob you of your mathematical brain.

So, you may ask, why hasn't Benezet's approach revolutionised maths teaching throughout the world, or at least throughout New Hampshire? Unfortunately, no history of this experiment has been written and the reasons for its abandonment in Manchester are not clear. However, this is clearly an experiment worth repeating.

Test : Do you have a mathematical brain?

I have included a self-test so that you can discover whether you think like a mathematician, that is, whether you can reformulate the problem so as to make it easier to solve. Here is a hint if you want to do well at it. Unlike the usual maths test, getting the right answer is not the aim of the game. It's how you get to it that is critical. The test items have mostly been taken from a range of interesting experiments on mathematical thinking, but I must stress that this test is not a properly standardised scientific instrument. It is designed, rather, to help you reflect on what you can achieve in maths.

This is different kind of test. Getting the right answer is not important. What is crucial is how you arrive at the answer. You will score points only for reasoning about the problem in sensible way. Try all the questions first. Then look up the answer.

Question 1.

The distance from Boston to Portland is 200 km. Three steamers leave Boston simultaneously for Portland. One makes the trip in 10 hours, one in 12 hours and one 15 hours. How long will it be before all are in Portland?

Question 2.

Estimate 3.21 x 5.04.

a. 1.6

b. 16

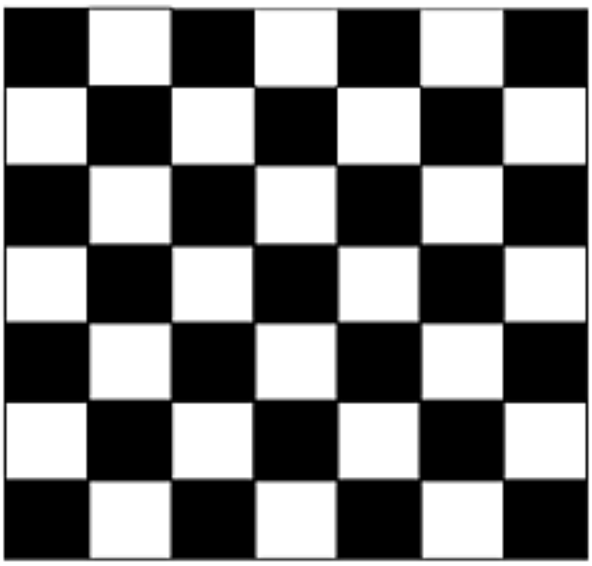
c. 160

d. 1600

e. no idea

Question 3.

How many black squares in this array?



Question 4.

Five hundred and sixty three soldiers need to be moved from Camp Alpha to Fort Baxter. Each bus can take 40 soldiers. How many busses will be needed for the journey?

Question 5.

$$\frac{273+273+273+273+273}{5} =$$

Question 6.

Jane's field is 270m long by 80m wide, while George's field is 280m long by 70m wide. Who has the larger field?

Question 7.

Find the sum of the numbers from 1-100.

Question 8.

Is it possible to colour faces of a cube with only three colours and none sharing an edge?

Question 9.

Imagine the following situation. I am standing in front of you holding a metre cube by the top and bottom corner. How many corners can you see (including the ones I am holding)?

Answers

Question 1.

If you attempted any addition, subtraction or multiplication of 10, 12 and 15, then no points for you. But if you picked 15 hours, 2 points.

> (This is a question that Benezet asked 9th grade (about 14 years old) children taught by the traditional method. Only 6 out of 29 got it right! He also gave this question to second grade children who, following his new curriculum, had not been taught formal arithmetic: they all got it right.)

Question 2.

1. If you attempted an exact calculation, no points.

2. If you estimated 16, then 2 points.

Question 3.

1. Count , 0 points

2. Multiply 7 x 7, divide by 2, and get 24.5. 0 points

3. Multiply 7 x7 and note that corners are black = 2 points

> (This question is taken from Chinn and Ashcroft's *Mathematics for Dyslexics*)

Question 4.

1. 14.075 = 0

2. 14 = 0

3. 15 = 2

(Obviously you cannot have 0.075 of a coach, and if you have rounded down, three soldiers would be left behind. Answers 1 and 2 show that your mathematical brain has been captured by the numbers).

Question 5.

1. If you added the 273s and then divided by 5, no points.

2. If you saw that there were five 273s, and reasoned that therefore the solution had to be 273, award yourself 2 points.

Question 6.

1. Calculated 270 x 80 (21,600) and 280 x 70 (19,600), and said that Jane has the bigger field.

2. Solved the problem without calculation. For example, by reasoning that Jane has 10 more big segments (10 x 270) while George has 10 more small segments (10 x 70). If you did this, you get 2 points.

3. If you solved the problem by drawing rectangles like this, then you also get two points.

(See Chapter 7 where I explained how even mathematics students make behave non-mathematically with this problem.)

Question 7.

1. You added 1+2+3+4+5+6+7+8+9+10 … 100, no points

2. You thought that the sequence could be folded on itself so that you could add 100+1, 99+2, 98+3, 97+4, 96+5 … so that you only had to do half the additions, and you then went on to carry out 50 additions, you get 1 point.

3. If you went one step beyond this, and saw that sum of the whole series was the sum of the first and last digits multiplied by half the number of digits, you may award yourself two points.

> (The young Gauss was asked a version of this question by an evidently very unmathematical teacher, who expected it to take the class the whole afternoon to solve, and was amazed when Gauss managed it after a few minutes thought.)

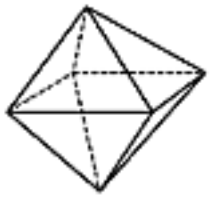
Question 8.

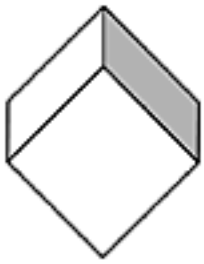
You have to imagine a cube with two opposite faces, say the top and bottom faces in the same colour, and the faces between them in alternating colours. If you got the correct answer without drawing, take 2 points; 1 point if you had to draw it first. And no points for the wrong answer. This is a test of spatial ability, which is important when thinking about geometrical aspects of mathematics.

Question 9.

Was your answer 5? If so, you imagined not a cube on its point, but an octahedron – two pyramids joined at the bottom.

A cube looks like this, with seven corners visible.





This problem is a stiff test of your logical construction of objects in space.

If you answered 7, then you get 2 points. If you drew a cube first, then just 1 point.

(My thanks to Geoff Hinton for this problem. I failed this test.)

Score

If you scored 18, then you have survived school with your mathematical brain intact. If you had special difficulty with the last two questions, then you have lost the ability to imagine objects in space. If you were trapped by the numbers in the first five questions, and carried out the counting or the calculations by force of habit, then I recommend you follow my three-step guide to mathematical sanity.

Building secure foundations and relieving anxiety

Here is my three-step programme for ridding yourself of maths anxiety.

Of course, I cannot give a you specific recipe since I don't know what level of mathematics you have reached. But the principles I will propose can apply at any level.

Step 1. Slow down

Much of maths anxiety comes from trying to do things too fast. Speed has always been stressed in maths tests. Children at the abacus juku are taught a procedure for solving a problem type, and simply practice that. Of course, they do get faster and more accurate. But because they are not taught alternative procedures, nor encouraged to develop them for themselves, abacus training will not help them see that there are alternatives. Professor Giyoo Hatano of Keio University, Tokyo, has found that abacus skills do not promote understanding.[8]

We have found that estimating the answer before calculating aids understanding, and we have used this method successfully with people who think themselves very poor at maths. This gets both sides of your brain working: the estimation processes of the right hemisphere and the sequential processes of the left hemisphere. It also gives you a check on your answer.

Step 2. Don't learn anything new, just try to understand what you think you already know

Try to find new ways to do old things. Take something really simple: 5 x 6. You probably "just know" the answer; you don't have to work it out. But take a moment to find another way to do it. For example, you could transform the problem into 10 x 3. This is very obvious, but it depends on an understanding that $n \times m = (n \times a) \times (m/a) = (n/a) \times (m \times a)$, or, in words, if you do one thing to one term and do the inverse to the other term, you won't affect the final answer. If you simplify, you will see that what is happening is that you are multiplying the whole problem by 1, because what you have actually done is multiply 5

8 Hatano (1997)

x 6 by 2/2. You can check that this rule holds by applying it to a new problem, e.g. 98 x 18. (Now there is a danger in the verbal formulation. You may think that because subtraction is somehow the inverse of addition, this problem is the same as 100 x 16 because you have added 2 to one term and subtracted 2 from the other. This is not the case, and the unconfident mathematician will find it very useful to see why.) Why not use a calculator to scale up the idea. If you were thinking through the implications of the 5 x 6 problem, you could test them out with very large numbers.

Doing all this may not help you not help you solve 5 x 6 faster, but it will help you understand multiplication, and how multiplication relates to division, subtraction and addition.

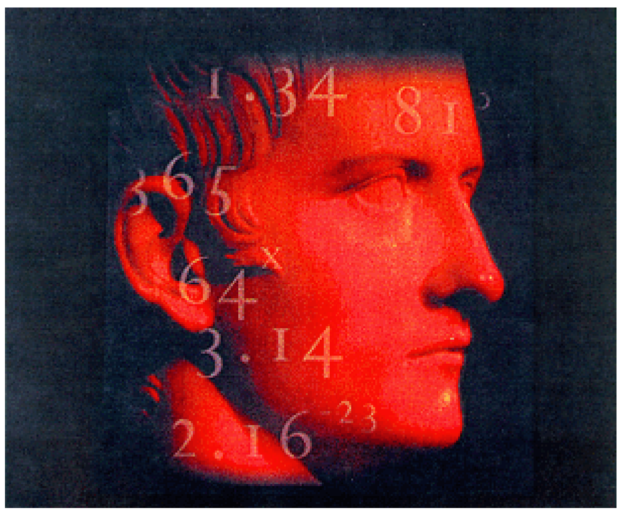
Step 3. Do it upside down

Don't repeat the same problems. Drill won't aid understanding. Try to see one problem from different perspectives. When you look at a familiar face standing on your head, or in a mirror, you will notice symmetries and asymmetries you had never noticed before. With mathematics, this can be achieved by trying to see a multiplication as an addition, or a subtraction, or combination of different operations. Do not worry about the answers, but about seeing mathematics and an integrated system. This is really a broader version of Step 2.

Follow these three steps when you are faced with a maths problem, and your worries will ease and eventually disappear. Good luck!

WHAT COUNTS

How Every Brain Is Hardwired For Math



Brian Butterworth

Preface   Reviews   Author   Interviews

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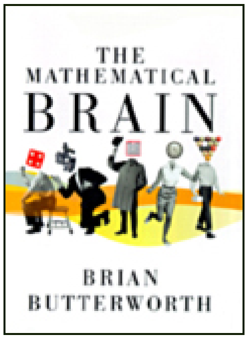

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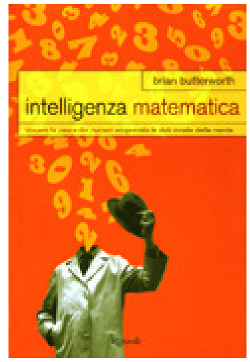

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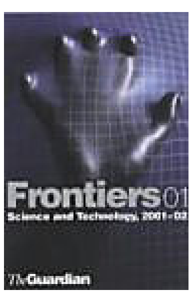

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