Mathematics and the Brain

Opening Address to The Mathematical Association, Reading: April 3rd 2002

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Thank you for inviting me. I am honoured to be addressing the Association.

I must start with some confessions. I am not a mathematician. I have never taught mathematics, except domestically. And, I am not going to treat the whole of mathematics in this address, but just one very basic element – number.

It is said to be good mental exercise to believe three impossible things before breakfast. It is now after lunch – whether this makes such a task easier or harder, I don't know, but I will be asking you to entertain three difficult, but not impossible, propositions.

First, that we are born with circuits in the brain that are specialised for number – in particular, for cardinal number – the number of objects in a set, the concept that underlies ordinary arithmetic.

The next two propositions follow from this one.

Second, if the genes can be programmed to build these specialised brain circuits, then now and again there will be a genetic anomaly and the brain does not grow the right circuits. People who suffer this condition are sometimes called dyscalculics. The main presenting symptom is that they have great difficulty understanding numbers. Being born with dyscalculia is a bit like being born colour-blind (also caused by a genetic anomaly) – except here it is a kind of number-blindness. The necessary neural system has just failed to develop properly. There is no cure, but there may be ways of working around the problem – and also ways of making it worse (as I will explain).

Third, if we are born with specialised number circuits, then we have evolved to understand at least some aspects of number that do not require the benefit of formal education. We may indeed have evolved to learn about the basic aspects of number in a way that is comparable to the way we learn a language. Formal instruction is not necessary, but exposure to a language is. Steven Pinker has argued that we have a language instinct that enables us to pick up our first language (or languages) quickly and easily in a language-rich setting. Perhaps the same is true for arithmetic. I will argue that early mathematics education fails to recognise our innate mathematical capacities, and, as a result, has turned millions of children away from mathematics. It is better, I will suggest, not to teach formal arithmetic at all – at least not in the primary school years. Let children pick it up, as they need it, as they are interested, just as they learn their native tongue.

Let me start with the first difficult proposition: that we are born with circuits in the brain that are specialised for number – in particular, for cardinal number. What is more, this system is the basis of subsequent mathematics learning – at least, subsequent arithmetic learning.

Number is just language

Surely, you will respond, There is nothing special in our brain for maths. You may counter with one of these plausible alternatives. Mathematics is a language, even the simple mathematics of numbers. Numbers are, after all, also words, "one, two, three". We are taught mathematics through language. Think of our multiplication tables. Didn't we have to recite them like poetry – or perhaps like the nonsense verse, *Jabberwocky* – until we knew them by heart as verbal formulae?

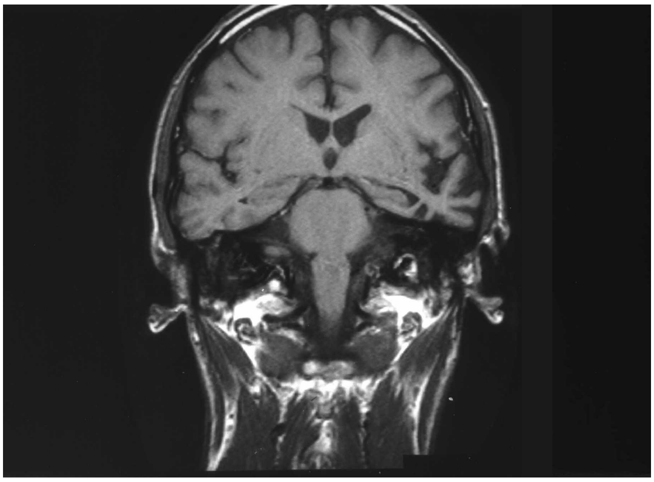
What is more didn't the mathematician, Keith Devlin, in his book *The Maths Gene*, demonstrate that the maths gene was in fact the number gene?

At a more sophisticated level, didn't the linguist, Noam Chomsky, argue that the essential feature of number was recursion – building larger and larger numbers recursively, even defining operations on numbers (such as addition), recursively? And that recursion is part of syntactic apparatus of language. So, if we have language, as a consequence of our language genes, we get recursion for free.

Counter arguments to language

One way to look at this is to ask whether people without language can manage to do mathematics. This where brain science provides critical evidence. One of our patients, who I shall call, Mr Harvey, a 64 year old former banker, was suffering from dementing illness that was slowly eating away at the language centres of his brain. When we started testing his calculating abilities, he could understand very little of what was said to him. He was unable to do one of the simplest tests of understanding – pointing to a named picture. He was also quite unable to name the commonest objects in a standard picture naming test. However, his arithmetic was near flawless

64 yr old, RH retired banker. Relatively preserved autobiographical memory; reduced vocabulary, many stereotyped phrases - "I delved into that ....". L. temporal atrophy increasing, some hippocampal and R temporal atrophy. Diagnosis of "semantic dementia".



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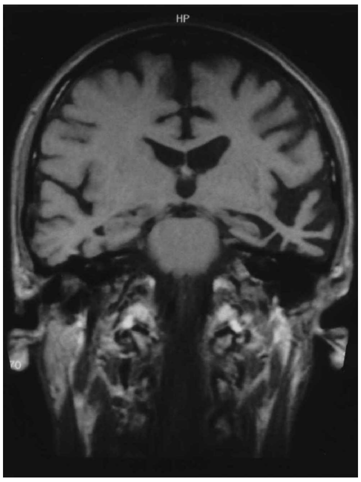


1998

Butterworth, B., Cappelletti, M., & Kopelman, M. (2001) *Nature Neuroscience*, 4, 784-786 and Cappelletti, M., Butterworth, B., & Kopelman, M. (2001) *Neuropsychologia*, 39, 1224-1239.

Graded Difficulty Naming Test	0/30
Naming line-drawings	
Vegetables	0/7
Body parts	2/8
Animals	0/9
Musical instruments	0/5
Furniture	1/5
Means of transport	1/6
Signs of the Zodiac	0/12
Naming real objects	0/15
Fluency (all categories)	0
Word-picture matching	chance

Tasks	%
Single Digit Arithmetic	
+	97
-	100
x	97
Multidigit Arithmetic	
Jackson & Warrington Test	96
+	95
-	100
x	100
Approximation	
Calculation	n.u
Placing Numbers on line	100
Transcoding	
From Numerals	100
To Numerals	100

So, language skills are not necessary for calculation. Nevertheless, it is possible that language is necessary for learning to calculate, but the ladder can be thrown away once you reach the landing of skilled performance. This is plausible, but there is one counter-example: A severely autistic young man, studied by Hermelin and O'Connor a few years ago. He was unable to speak, to understand speech, and he could communicate in only the most rudimentary way with gestures. He was nonetheless able to find factors and primes in numbers faster and more accurately than a university-trained mathematician (and now FRS). We don't know how he learned.

More striking, perhaps, is that infants, even in the first weeks of life – long before they are able to understand speech or speak themselves, have simple numerical abilities. They notice, for example, when the number of objects in a display changes. They will look longer at a new display with more or fewer objects, than one with the same number. What's more infants can add and subtract: if you show them one doll going behind a screen and then a second doll going behind the screen, they will expect there to be two dolls when the screen is removed. We know this because they look longer when there is one doll or three dolls – they look longer at the unexpected. Similarly, if they see two dolls go behind the screen and then one is removed, they will expect there to be one left.

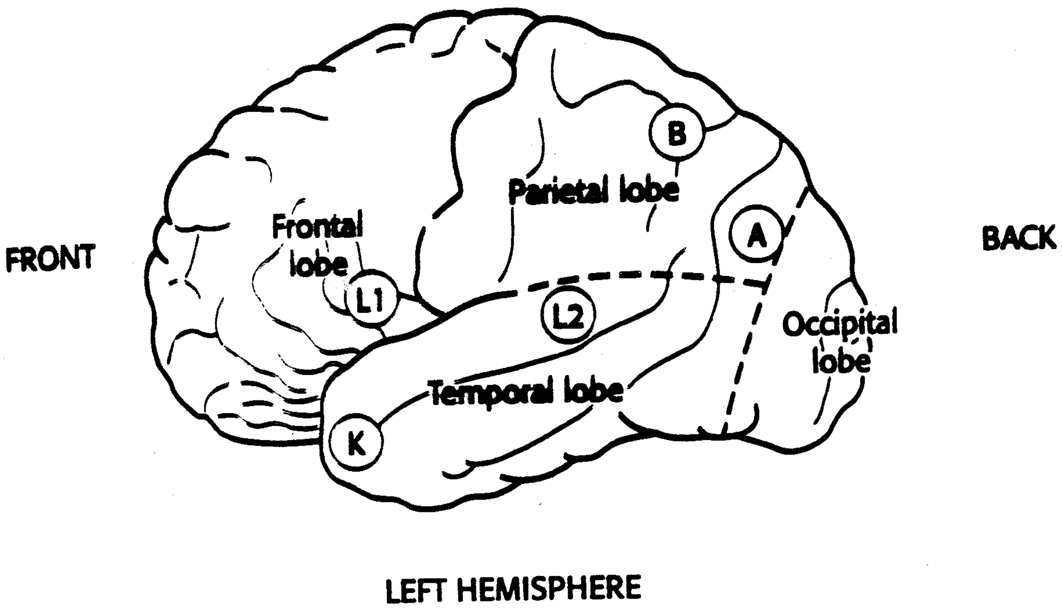
Here are simple things that can be done without language. We do not, at present, understand the relationship between these infantile capacities and later arithmetic skill. My own guess, and this is only a guess, is that such capacities form the basis of later skill. Being able to recognise small numerosities may support learning to count – it's a way of checking the accuracy of the count.

However, we do not know yet which part of the brain the babies are using for these tasks. We now know which parts of the brain adults use, and for this innate system to serve as the basis for later developments it should be the same.

Not only is language not necessary for simple numerical tasks, it is also not sufficient. Again we can draw on evidence from patients. A patient I shall call Signora Gaddi kept the books of the family hotel in NE Italy before she suffered a stroke. This left her language entirely intact, but her numerical skills profoundly compromised. For example, she was unable to count above four. If you present five objects for her to count, she would say. “uno, due, tre, Quattro, la mia matematica finisce qui.” (“My mathematics finishes here.”) We tried her on all kinds of tasks – adding, subtracting, and so on – but she couldn’t get above four on any of them. Even asking her to say whether 5 or 10 were larger. She had lost her numbers above four completely – she could not even say the number of days in the week, her age, shoe size or address. It was very seriously handicapping. She needed a special device for telephoning people in case of an emergency since she could no longer dial the numbers herself.

Actually it is not surprising that language and number should dissociate in this way. As you can see from the next picture, they use very different parts of the brain.

Language and Mathematics areas in the brain.



Language:	Mathematics:	General knowledge:
L1 Brocas area; L2 Wernicke's area	A Inferior parietal lobe; B Intraparietal sulcus	K

(From Butterworth, B (1999)
The Mathematical Brain. London: Macmillan)

Number as logic

There is another line you might take against my claim that there are special number circuits. You may claim that really arithmetic is nothing more than logic. A distinguished predecessor to address this association, Alfred North Whitehead, famously argued this in his work with Bertrand Russell, *Principia Mathematica*. We now know that their proof was faulty, and that, as Kurt Gödel showed, it is impossible to derive arithmetic from logic alone. However, you may say that, well, perhaps we cannot have a complete proof, but surely maths and logic are closely linked, and that from a practical point of view the ability to reason is fundamental to doing arithmetic and more advanced mathematics.

You may cite the work of the celebrated Swiss psychologist, Jean Piaget, who was influenced by the logicism of Russell, Whitehead and Frege, and invited mathematicians and logicians to attend his seminars in Geneva. He claimed that the development of certain logical concepts and abilities was indeed necessary, and sufficient, - "prerequisites" for the acquiring the idea of number – in fact of cardinal number. He argued that the child must understand the concept of classes and class inclusion, and must be able to reason transitively. This occurs around the age of four in most children. Children who possess the concept know that simply moving objects around doesn't alter the cardinality of a set – an object has to be added or taken away. This is Piaget's famous demonstration of the "conservation of number". This where there are two equal rows of objects, say 10 disks and 10 cubes lined up in one-to-one correspondence. The experimenter asks the child are there more disks or cubes. The child says, correctly, they are the same. Then the experimenter spreads out the disks, and asks the question again. The child who does not yet possess the "number concept" will say that there are more disks. This spatial transformation seems to be believed by the child to affect the cardinality of the set.

We have seen that infants, and even some other species seem to have a sense of number, and obviously are unable to carry out transitive reasoning or class inclusion tasks. Does this mean that Piaget was wrong? Several authors, including myself, have said so. However, I now think differently. Perhaps one shouldn't think of a single concept of cardinality, but rather increasingly general concepts – up to infinite cardinals. Now it could be that infants and ravens have a limited sense of cardinality – including the idea that objects have to be added or taken away to change cardinality – but there is a low upper bound on the actual cardinalities they can handle – about 4. Above that, numbers become vague, and it is doubtful that their concept ranges over all types of objects. It must be said that Karen Wynn has demonstrated that 6 month olds seem to have a cardinal concept that ranges over actions (the number of jumps a doll makes) as well as physical objects.

My approach is rather straightforward. Can one find a person with the full range of Piagetian prerequisites nevertheless have no concept of number? Remember Signora Gaddi. We tested her on Piagetian type tasks, and she performed flawlessly on all of them. But, as we have seen, she only had number concepts up to 4.

So being able to reason logically isn't sufficient, but is it necessary? Let me tell you about a 86 year old Alzheimer patient studied by a team of neuropsychologists in Belgium led by Xavier Seron. I will call him Monsieur Van. M. Van failed reasoning tests that Piaget claimed were a prerequisite for having the concept of number, including the conservation of number test – a test most four-year olds can manage. But his normal calculation was fast and accurate. What is more he could tell which three and four digit numbers were squares – 4096 or 4099. He could select the root of four digit numbers from four alternatives, e.g. 3844: is it 42, 61, 62, or 68?

Now most neuroscientists would not be surprised at this dissociation between logical reasoning and numerical abilities: different brain systems are involved. Logical reasoning is a frontal lobe function, while calculation is the function of the parietal lobes.

This is why one can find these dissociations: Signora Gaddi has damage to her parietal lobes while M. Van has damage to his frontal lobes.

Now I have shown that numerical abilities involve a special brain area, but, as you have realised, I haven't shown that we are born with these brain areas already specialised for number. Which brain areas do the babies use when they are having these arithmetical expectations? We don't know. And therefore we don't know whether we build upon them as we acquire more and more numerical skills.

What about animals? If we have inherited ancestral capacities then homologous brain areas in our monkey cousins should be active when they are doing numerical tasks. One paper on monkeys was published in Nature a couple of weeks ago showing that in one task that involves pushing a lever five times, some parietal lobe cells appeared tuned to this number, and would fire only when the monkey was waiting to push the lever. But, this is just one number, and a lot of training was required to get the monkey to do the task at all – 10 months.

So, this is why I said at the beginning that I was asking you to believe difficult propositions. The proposition that we are born with number-specific brain circuits has not been demonstrated, as you will have gathered. You may be persuaded the dissociations between patients and in-brain-imaging studies mean that there are number-specific circuits; you may even be persuaded that without them there will be difficulties learning maths; and you may be persuaded that infants are born with simple numerical capacities. However, you will have noticed two big gaps in my argument. First, I have not been able to provide evidence for the relationship between infant capacities and later skills or indeed this specialised brain area. We have no idea which brain system underlies these infant capacities. Second, if we are born with these circuits, where are the genes that build them? Again, I have to admit that we do not yet know.

On balance, though, it seems to me that we have enough converging lines of evidence to makes such a proposition currently the best bet.

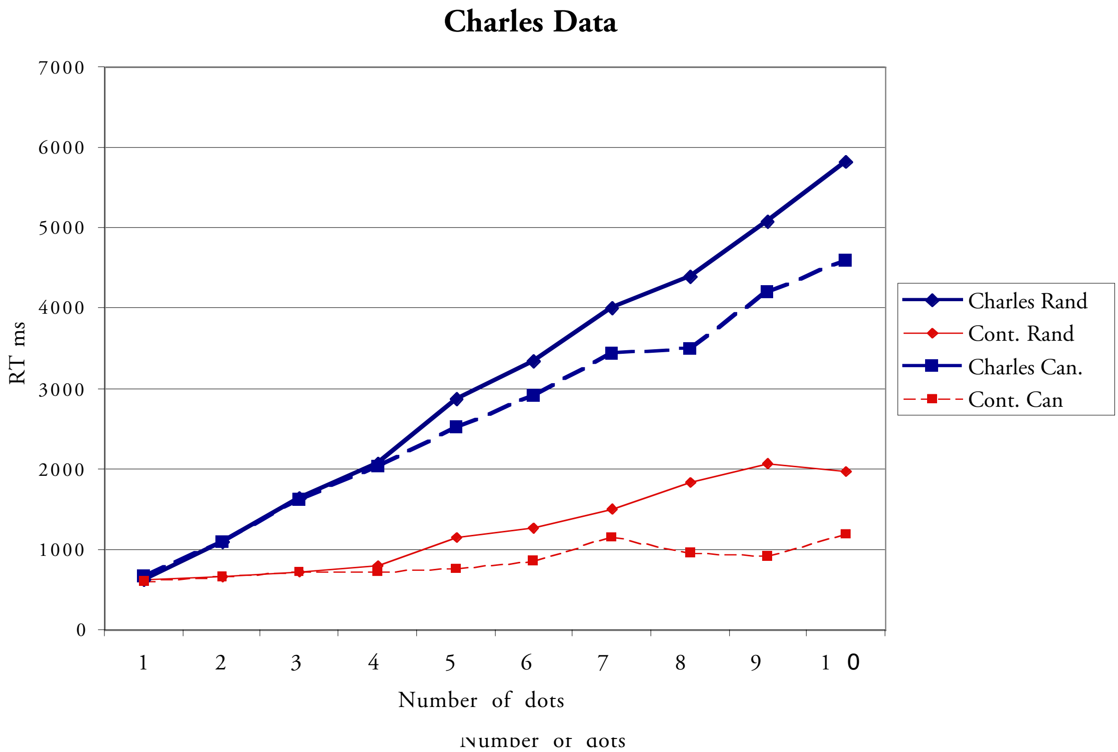
Dyscalculia

Now if we have genes for basic number capacities, as we have genes for seeing the world in colour, then some people will have a genetic anomaly, a mutation, that means that they will lack this capacity – just as there are people who cannot see the world in colour in the normal way. We actually know what gene locus is responsible for colour vision, though, as I have said, we do not yet know about the number gene.

However, we now know there are people who seem to suffer from a kind of number-blindness, called “dyscalculia”. Let me tell you about one of them.

I first met Charles (not his real name), when he was 30 years old, and proud possessor of a degree in psychology. Getting a degree was an achievement, but entry to university in the first place was even more impressive, since he had, despite his best efforts, failed the normal condition for entry, Maths G.C.S.E.

Charles is hard-working and intelligent, but his poor number skills have always been a severe handicap. Shopping is a constant embarrassment: he doesn’t understand prices, and has no idea of the total cost of his shopping basket. When he comes to the till, he has no idea of how much money to tender or whether the change is correct. Charles, we discovered, added and multiplied using his fingers, and was unable to do two-digit written arithmetic problems such as 37-19. What really surprised us was that he couldn’t tell that 9 was bigger or smaller than 3, and had to use his fingers to work it out. In fact, he wasn’t deficient just in arithmetic achievement – for which there may be many contributing factors – but also on the simplest numerical tasks, tasks which do not need much learning at all.


Charles Data
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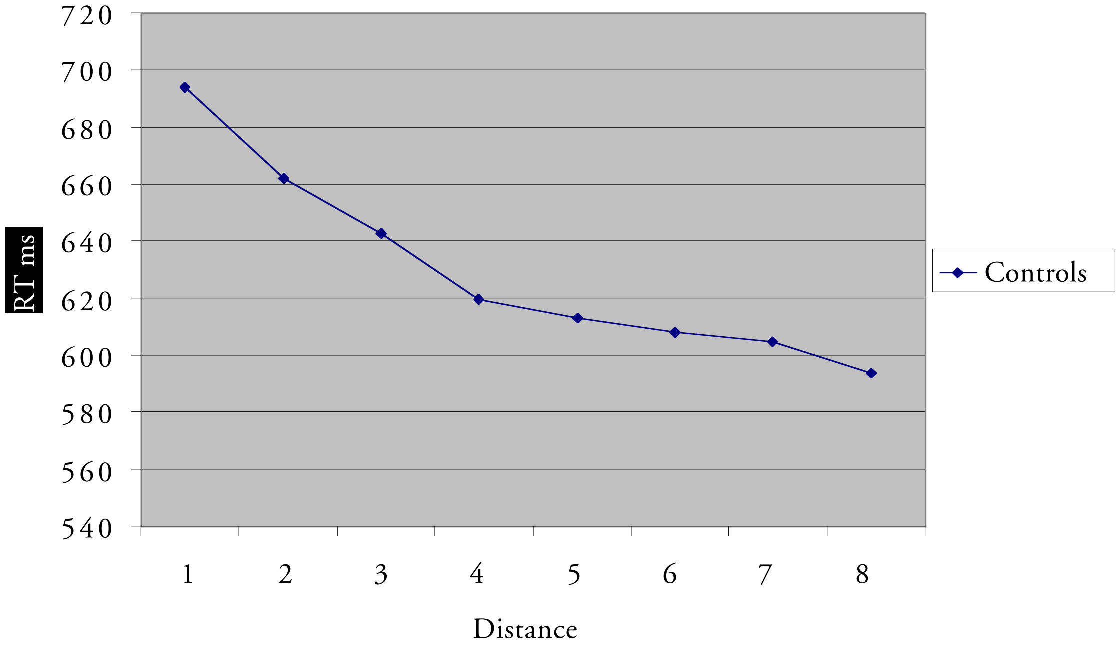
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Charles Data

Charles is an example, a severe example, of a condition known as “dyscalculia” that affects the ability to acquire arithmetical skills. Dyscalculic learners may have difficulty understanding even very simple numerical ideas. They may find the daily maths lesson a source of enormous anxiety since they struggle to understand what is obvious to all their classmates.

Unfortunately, dyscalculia is not widely recognised. For dyscalculics, the situation is rather like that for dyslexics 30 years ago. Teachers, parents, the world at large, think they must be stupid not to understand ideas and methods easily acquired by the rest of us. Some people may regard the label “dyscalculia” the kind of excuse middle-class parents make for their underachieving children, just as people used to regard the label “dyslexia”.

The DfES define dyslcalculia as, “A condition that affects the ability to acquire arithmetical skills. Dyscalculic learners may have difficulty understanding simple number concepts, lack an intuitive grasp of numbers, and have problems learning number facts and procedures. Even if they produce a correct answer or use a correct method, they may do so mechanically and without confidence.”

This captures what many dyscalculics, like Charles, feel about maths. It is incomprehensible.

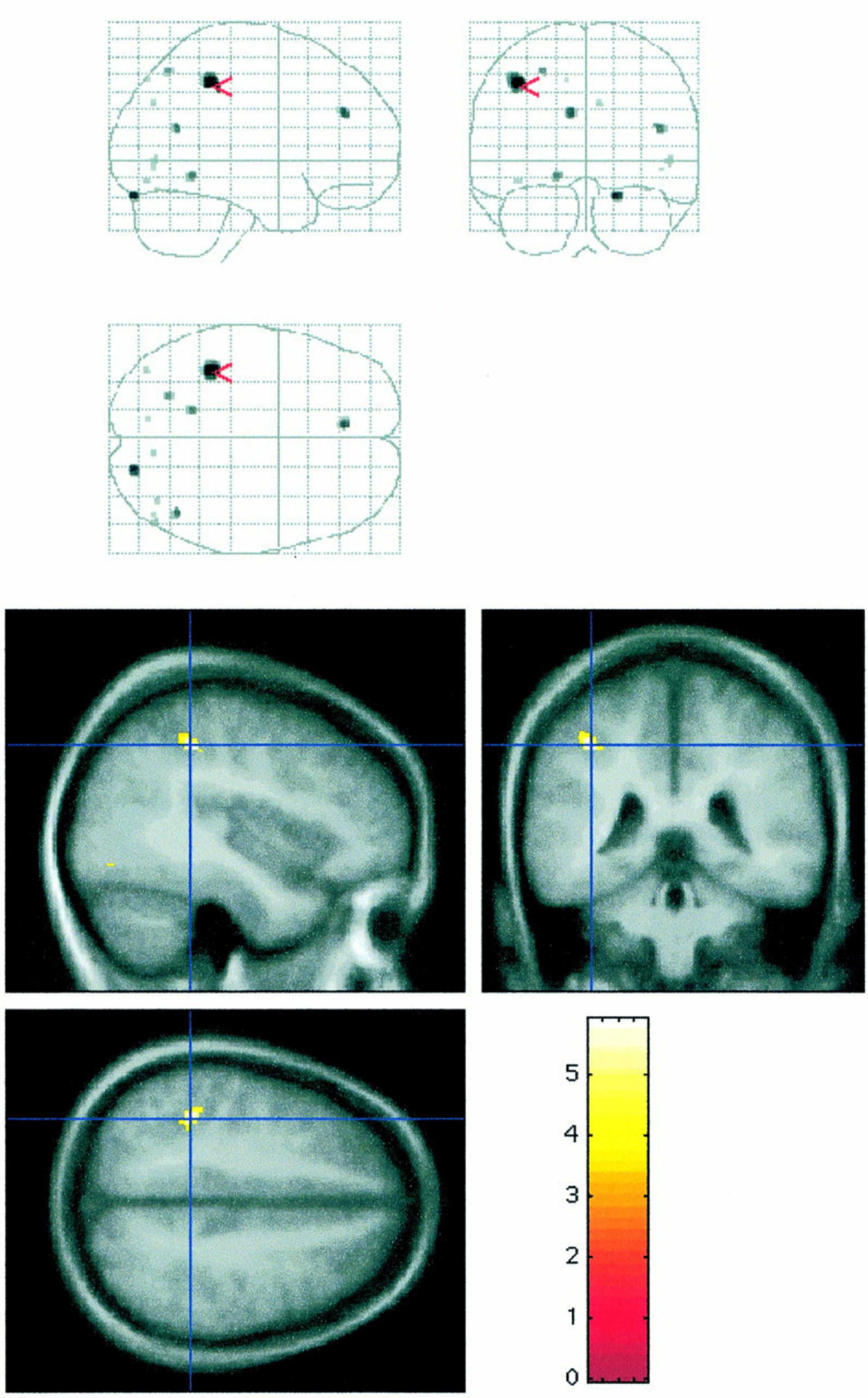
How widespread is this problem? The best current estimates suggest that about 5-6% of children of average to superior intelligence will have a real specific learning deficit for maths. This is a similar prevalence to dyslexia. So, dyscalculia is a big problem not just for individuals who suffer from it, but for the nation.

Dyscalculia seems to be particularly rife among dyslexics, with around 40% of children with reading difficulties also having difficulties in learning maths. This is a double whammy for them. It is also a serious puzzle for science. After all, the other 60% have no more problems than normal. Indeed, dyslexics can be outstanding mathematicians. What is the difference between those dyslexics who do suffer from dyscalculia and those who do not? What is it about dyslexics that puts them at risk of dyscalculia at all?

On the other side of the equation, something like 1-2% of children with no reading difficulties, and with normal cognitive abilities, are two years behind their peers. So, although there is an association between dyslexia and maths learning difficulties, the latter can occur alone.

Although there has been little systematic work on either the causes or the varieties of dyscalculia, it is clear that this condition is real and a real handicap. Fortunately, the National Numeracy Strategy has, after a little persuasion, issued guidance to schools about dyscalculia. Ours is perhaps the first government anywhere to officially recognise dyscalculia.

Do we have any idea what is wrong with the brains of dyscalculics? Some very suggestive evidence was published last year by Elizabeth Isaacs and her team at Great Ormond St Hospital. They compared the brains of two groups of adolescent children. Both groups were of normal intelligence, but one group performed very poorly on a standard test of simple arithmetic. The brains of this group differed from the other group in one respect: they had less grey matter in the left parietal lobe. In fact, in a small area of the parietal lobe we know from other studies to be crucially involved in calculation!



Brain area (yellow) where there is less grey matter in adolescents with poor number skills.
(From Isaacs et al, *Brain*, 124, 1701-1707 (2001)

Believing in dyscalculia was, perhaps, not so difficult.

Benezet's experiment

My next proposition may be very hard to swallow and digest.

It is that it is better not to teach formal arithmetic at all – at least not in the primary school years. Let children pick it up, as they need it, as they are interested, just as they learn their native tongue.

Of course, this proposition goes against all current educational practice and theory: the earlier you start teaching the better. Children are introduced to the rudiments of reading and arithmetic in nursery school, when they could be playing. Does it really help? I was reading about some new research that demonstrates that foetuses in the womb can learn, yes learn, to discriminate between vowel sounds. Will ambitious parents start trying to teach their unborn offspring useful language skills so they will be ahead of the peers – their competitors – from the moment they emerge into the world?

Now, there is indirect evidence that when it comes to arithmetic an early start – not intrauterine, admittedly - doesn't help. In Europe children start school at different ages – from 5 to 7 – and there is no obvious correlation between age of entry to schooling and performance in international comparisons such as TIMSS or the PISA study.

Other evidence is that children pick up a lot of number skills before they go to school.

Children will have a sense of cardinality. They can match two collections of objects by number; they can tell which of two collections has more objects in it; they can share (divide) a collection into equal subcollections; they understand which transformations of a collection will change its cardinality (e.g. adding or subtracting objects) and which will not; of course, in our numerate society, they will also be able to count. This is a much more complex achievement than it might appear to those of us who have been counting for years.

The child has to learn a special collection of words – one, two, three.

She has to learn to keep them in the same order in all counts; and come to keep them in the conventional order (this usually takes two steps: fixed order, conventional order).

She has to learn how to match each counting word to each object in the collection.

She has to learn that any object, in any modality or none, is countable – there can be three rabbits, three noises, or three wishes, in a countable collection.

And, the child must come to understand that there is no limit to counting.

Many children will be able to add and subtract small numbers before they come to school.

In cultures where children are not formally educated, but have to participate in a numerate culture – e.g. illiterate market traders – they quickly develop their own techniques of calculation. For example, one young Brazilian trader described by Terezinha Nunes, was asked the the price of 10 oranges at 35 cruzados per orange. He worked it out thus: three oranges are 105; so three of those is three hundred and fifteen; and one more, that three hundred and fifty. Nunes notes that when these children go to school and are taught formal methods, their performance gets worse!

But I can see you are not completely convinced by this. What you want, if I am right, is a properly controlled experiment: children matched (say, by background and intelligence), half being taught arithmetic traditionally and half not taught arithmetic at all for the whole of the primary years. You would like external assessment of the outcomes, would you not?

Is it conceivable that any school, any local education authority, would countenance such an "experiment"? It is certainly not conceivable today in Britain. The *Daily Mail* and its middle-class parent readers would be apoplectic. Tony wouldn't dare. I doubt that *Guardian*-reading parents would be much keener, either.

But the experiment has been done. Not here. And not recently.

The experiment took place in Manchester, New Hampshire, in the 1930's. The superintendent of schools Louis P. Benezet reasoned as follows *:

> In the first place, it seems to me that we waste much time in the elementary schools, wrestling with stuff that ought to be omitted or postponed until the children are in need of studying it. If I had my way, I would omit arithmetic from the first six grades. I would allow the children to practise making change with imitation money, if you wish, but outside of making change, where does an eleven-year-old child ever have to use arithmetic?
>
> I feel that it is all nonsense to take eight years to get children thru the ordinary arithmetic assignment of the elementary schools. What possible needs has a ten-year-old child for a knowledge of long division? The whole subject of arithmetic could be postponed until the seventh year of school, and it could be mastered in two years' study by any normal child.
>
> For some years, I had noted that the effect of the early introduction of arithmetic had been to dull and almost chloroform the child's reasoning faculties. There was a certain problem which I tried out, not once but a hundred times, in grades six, seven, and eight. Here is the problem: "If I can walk a hundred yards in a minute [and I can], how many miles can I walk in an hour, keeping up the same rate of speed?"
>
> In nineteen cases out of twenty the answer given me would be six thousand, and, if I beamed approval and smiled, the class settled back, well satisfied. But if I should happen to say, "I see. That means that I could walk from here to San Francisco and back in an hour" there would invariably be a laugh and the children would look foolish.

Benezet had been concerned not only that the standard of maths had been very disappointing in his school district, but the children's ability to express themselves in speech and writing was

*http://www.inference.phy.cam.ac.uk/sanjoy/benezet/

depressingly poor. With less time wasted on long division, more time could be spent reading, writing, and thinking. He wrote,

> In the fall of 1929, I made up my mind to try the experiment of abandoning all formal instruction in arithmetic below the seventh grade and concentrating on teaching the children to read, to reason, and to recite - my new Three R's. And by reciting I did not mean giving back, verbatim, the words of the teacher or of the textbook. I meant speaking the English language. I picked out five rooms - three third grades, one combining the third and fourth grades, and one fifth grade.

Teachers in these experimental rooms were not to teach arithmetic, but should give the children practice in estimating heights, lengths, areas, distances, and the like. Starting in 1932, children in the experimental rooms received no arithmetic teaching until 6th grade.

How did these experimental children fare under the new curriculum? We have two sources of evidence: Benezet's own method of assessment, which was to go to a room, accompanied by a stenographer, set the class a problem and record their answers verbatim. The second method used formal tests of mathematical achievement, along with IQ tests, given at three points in the sixth grade – on entry, after three months and after eight months. These were carried out by Etta Berman, a maths teacher, as part of her Master of Education thesis at Boston University, supervised by Guy Wilson, a leading expert in maths education and the deviser of standard tests then widely used and employed in this study.

Let me give an example of Benezet's method.

Benezet Regular Group

> "Here is a wooden pole that is stuck in the mud at the bottom of a pond. There is some water above the mud and part of the pole sticks up into the air. One-half of the pole is in the mud; 2/3 of the rest is in the water; and one foot is sticking out into the air. Now, how long is the pole?"

> First child: "You multiply 1/2 by 2/3 and then you add one foot to that."
>
> Second child: "Add one foot and 2/3 and 1/2."
>
> Third child: "Add the 2/3 and 1/2 first and then add the one foot."
>
> Fourth: "Add all of them and see how long the pole is."
>
> Next child: "One foot equals 1/3. Two thirds divided into 6 equals 3 times 2 equals 6. Six and 4 equals 10. Ten and 3 equals 13 feet."
>
> You will note that not one child saw the essential point, that half the pole was buried in the mud and the other half of it was above the mud and that one-third of this half equalled one foot. Their only thought was to manipulate the numbers, hoping that somehow they would get the right answer. I next asked, "Is there anybody who knows some way to get the length?"
>
> Next child: "One foot equals 3/3. Two-thirds and 1/2 multiplied by 6."
>
> My next question was, "Why do you multiply by 6?"
>
> The child, making a stab in the dark, said, "Divide."

Then Benezet took the problem to a room that used the new curriculum. Here is what happened:

Benezet Experimental Group

> First child: "You would have to find out how many feet there are in the mud."
>
> "And what else?" said I.
>
> Another child: "How many feet in the water and add them together."

> "How would you go to work and get that?" said I to another child.
>
> "There are 3 feet in a yard. One yard is in the mud. One yard equals 36 inches. If two-thirds of the rest is in the water and one foot in the air [one foot equals twelve inches] the part in the water is twice the part in the air so that it must be 2 feet or 24 inches. If there are 3 feet above the mud and 3 feet in the mud it means that the pole is 6 feet or 72 inches long. Seventy-two inches equals two yards."
>
> It amazed me to see how this child translated all the measurements into inches. As a matter of fact, to her, the problem was so simple and was solved so easily, that she could not believe that she was doing all that was necessary in telling me that the pole was 6 feet long. She had to get it into 72 inches and two yards to make it hard enough to justify my asking such a problem.
>
> The next child went on to say, "One-half of the pole is in the mud and half must be above the mud. If two-thirds is in the water, then two-thirds and one foot equals 3 feet, plus the 3 feet in the mud equals 6 feet."
>
> The problem seemed very simple to these children who had been taught to use their heads instead of their pencils.

Berman paired 34 girls and 27 boys from 8 rooms in the regular and experimental groups, matched for intelligence on tests used at that time (Kuhlman-Anderson and Haggerty) and approximately for social background (though from the fact that more experimental mothers, worked, Berman concludes they may have been of lower social background). What Berman doesn't control for is parents' first language. Benezet realised that he was going to have a better chance of trying this experiment with first-generation immigrant families, and he guessed that only one in ten had English as their language at home.

Berman's Stats from 1933

(when the programme had just begun)

	Entry	**Exp**	**Reg**	**Reg adv**
Dec - Jan				
+	82	92	93	1
-	48	88	94	6
x	27	80	89	9
÷	18	73	92	19
÷ long	0	60	91	24
March				
+		47	48	0
-		45	48	3
x		36	42	6
÷		37	44	7
fractions		15	33	18
May				
+		46	47	1
-		46	46	0
x		36	39	3
fractions		41	44	3
		23	31	8

It is clear that when the experimental children entered 6th grade, they were not as good at the formal tests as the regular children. But look how quickly they caught up. In fact, one of the experimental rooms was the best in the city.

So what had the experimental children been doing while the regular children were being drilled in arithmetic? Benezet's three Rs – reading, reciting and reasoning. There was quite a lot of number work, though no formal instruction, and no specific period in which arithmetic was turned. Rather the teacher responded to the number topics pupils themselves brought up. The new curriculum included teaching the children to recognise and read the numbers up to 100, dates and times, terms like "larger", "smaller", "twice", and money terms (Grades 1 and 2), measurement (Grades 3 and 4), and more on estimating (a constant theme) in grade 5.

The result of focussing on the three Rs was that their spoken and written works was far more interesting and imaginative. They used a wider range of words, and their spelling was better! And, of course, they reasoned about arithmetical problems in a sensible way!

What is the consequence of all the drill that so disfigures education here and in the US?

Here is one example, from a doyen of US maths education, Alan Schoenfeld: "What's all the fuss about metacognition", pp. 195-6, in *Cognitive Science and Mathematics Education*, Alan Schoenfeld, ed. (Lawrence Erlbaum, 1987)

> One of the problems on the NAEP secondary mathematics exam, which was administered to a stratified sample of 45,000 students nationwide, was the following: An army bus holds 36 soldiers. If 11,28 soldiers are being bused to their training site, how many buses are needed?
>
> Seventy percent of the students who took the exam set up the correct long division and performed it correctly. However, the following are the answers those students gave to the question of "how many buses are needed?": 29% said…"31 remainder 12"; 18% said…"31"; 23%

said…“32”, which is correct. (30% did not do the computation correctly).

It’s frightening enough that fewer than one-fourth of the students got the right answer. More frightening is that almost one out of three students said that the number of buses needed is “31 remainder ”.

My three difficult propositions were

1. Most of us are born with a specific innate capacity for cardinal number.

2. A few of us are not. These are the dyscalculics. This is a serious problem for educationalists, and for society in general.

3. The rest of us did not need hours and hours of formal instruction in arithmetic until we are 11. In an informal and number-rich environment such as Benezet’s classrooms, we can pick up what we need, as easily and as naturally, as picking up the words we need. All that drill choloroforms the mind so that its victims are incapable of reasoning properly about numerical problems.

Let our children go!